Dynamic Modeling and Analysis of a Synchronous Generator in a Nuclear Power Plant

Attila Fodor, Attila Magyar, Katalin M. Hangos

***Abstract*— A simple dynamic model of an industrial size synchronous generator operating in a nuclear power plant is developed in this paper based on first engineering principles. The constructed state-space model consists of an LTI state equation and a bi-linear output equation. It has been shown that the model is asymptotically stable with parameters obtained from the literature for a similar generator but it is very close to the stability boundary.**

The effect of load disturbances on the partially controlled generator has been analyzed by simulation using a traditional PI controller. It has been found that the controlled system is stable and can follow the set-point changes in the effective power well. The disturbance rejection of the controller is also satisfactory.

I. INTRODUCTION

Nuclear power plants are important electrical energy providers worldwide. Their efficient and safe operation is of great practical importance that is a subject of numerous studies in the literature. These include control oriented studies where a usual aim is to re-design or retrofit a controller or the control system of an existing nuclear power plant or part of it (see e.g. [2], [3]). For controller (re)design purposes simple low dimensional concentrated parameter models are needed that can be constructed from first engineering principles, see e.g. [4], [5].

Nuclear power plants generate electrical power from nuclear energy, where the final stage of the power production includes a synchronous generator that is driven by a turbine. Similarly to other power plants, both the effective and reactive components of the generated power depend on the need of the consumers and on their own operability criteria. This consumer generated time-varying load is the major disturbance that should be taken care of by the generator controller.

In Figure 1 the time varying output of a nuclear power plant, the Paks Nuclear Power Plant (NPP) in Hungary, is depicted during load changing transients. It can be seen that the reactive power is also changing in parallel to the effective power. Since a unit in our NPP contains two generators, the same signals belonging to the two generators are both depicted.

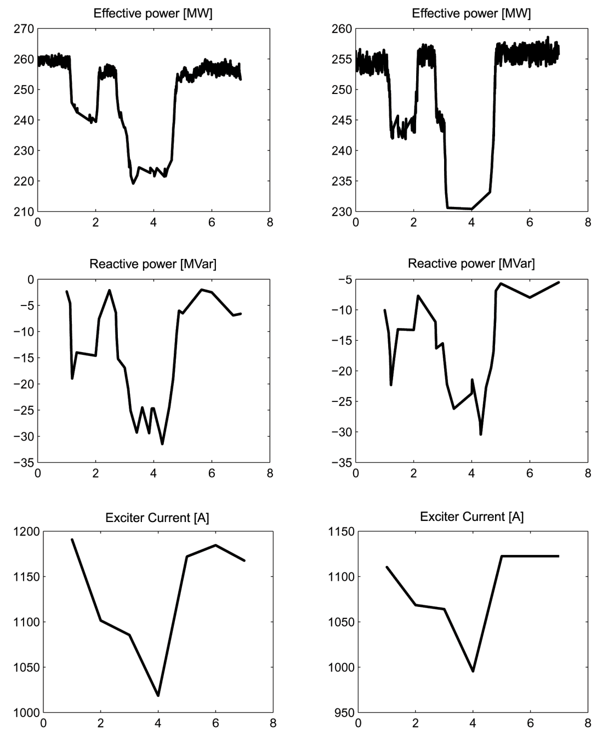

Fig. 1. Effective and reactive power and the exiting current of the generators during load changing transients

It can be seen in Figure 1 that the reactive powers of the synchronous generators are changing during the power switching.

Because of the specialities and great practical importance of the synchronous generators in power plants, their modeling for control purposes is well investigated in the literature. Besides of the basic textbooks (see e.g. [1]) that describe the modeling and use the developed models for the design of various controllers [6], [7]. These papers, however, do not take the special circumstances found in nuclear power plants in to account and it results in special generator models. The aim of this paper is to propose a simple dynamic model of a synchronous generator in a nuclear power plant for control studies.

This work was supported by the Hungarian Research Fund through grant K67625

A. Fodor is with Department of Electrical Engineering and Information Systems, University of Pannonia, 8200 Veszprém, Hungary foa@almos.vein.hu

A. Magyar is with Department of Electrical Engineering and Information Systems, University of Pannonia, 8200 Veszprém, Hungary amagyar@almos.vein.hu

K. M. Hangos is with Department of Electrical Engineering and Information Systems, University of Pannonia, 8200 Veszprém, Hungary, and with the Process Control Research Group, Computer and Automation Research Institute HAS, Budapest, Hungary hangos@sztaki.hu

II. The basic power generating equipments of the Paks Nuclear Power Plant

The Paks NPP is a pressurized water nuclear power plant that has four power generating units.

A. Overview of the Plant

The pressurized water nuclear power plants always have two separated coolant circuits, the radio-active primary coolant circuit and the secondary coolant circuit. The main function of the primary coolant circuit is to cool the reactor and heat the stream generators. Each nuclear unit applies over-pressure water cooling (120 bar, 297^o) where the cooling liquid serves as moderator, too. Every nuclear unit operates six stream generators with capacity 450 t/hour on temperature 260^oC and pressure 46 bar. These six steam generators are connected to two turbo-generators, three to each. The flowsheet of the secondary circuit is shown in Figure 2.

Every nuclear unit operates two independent secondary coolant circuits, that are controlled by independent control systems. The secondary coolant circuits begin with the three stream generators that provide stream for the turbines. There are three turbines in each secondary coolant circuit (one high pressure and two low pressure ones), attached to one synchronous generator and one exciter machine set on the same axis.

B. The Turbogenerator

The turbo generator, the subject of our study, is a special synchronous generator with a special cooling system. The armature has been cooled by water and the rotor has been cooled by hydrogen. The used generators have been designed and have been built by the Hungarian Ganz Ltd. at the beginning of the 1980s. The original nominal engineering data of the generators:

Apparent energy	259 MVA,
Active power	220 MVA,
$\cos\varphi$	0.85,
Voltage	15.75 KV,
Current	9490 A,
Frequency	50 Hz,
Speed	3000 1/min,
Excitation current	1450 A,
Excitation voltage	435 V.

In the Hungarian nuclear power station the exciter field regulator of the synchronous generator currently does not control the reactive power only the effective power. *The final aim of our study is to design a controller that can control the reactive power such that its generation is minimized in such a way that the quality of the control of the effective power remains (nearly) unchanged.*

There are three generator exciter field regulators for each generators (automatic, manual, and the back-up). The manual generator exciter field regulator performs output voltage control of the synchronous generator by applying a sequential control to the output voltage of generator that is constrained by a voltage limiter. The sequential controller is a PI controller.

The generated effective and reactive power are not independent but they are related by $S^2 = P^2 + Q^2$ and $S = \sqrt{3}UI$, where P is the effective and Q is the reactive power. If we use the generator at the nominal operating point S the reactive power can be 136.7 MVA, but if we want to increase the effective power of the generator we have to decrease the reactive power. The efficiency of the primary and secondary coolant circuits has recently been increased with constant reactor heat powers, so the output power will be about 500 MVA for each unit. At the same time, the reactive power of the generator can be 67.7 MVA as a maximum, therefore we have to design a new controller for the synchronous generator, which controls both the effective and the reactive power accordingly. Note that there is no reactive power control for the generators at present.

Before we start planning the controller, we have to build, verify and validate the model of the synchronous generator.

III. The Model of the Synchronous Generator

In this section the state-space model for a synchronous generator is described that will be used for stability analysis and controller design [1].

A. The Engineering Model

For constructing the synchronous generator model, let us make the following assumptions:

- a symmetrical tri-phase stator windings system in machine is assumed,
- one field winding in machine is considered,
- there are two amortisseur or damper windings in machine
- the copper losses and the slots in the machine are neglected,
- the spatial distribution of the stator fluxes and apertures wave are considered sinusoidal,
- stator and rotor permeability are assumed to be infinite.

It is also assumed that all the losses due to wiring, saturation, and slots can be neglected.

The six windings (three stators, one rotor and two damper) are magnetically coupled. The magnetic coupling between the the windings is a function of the rotor position. Thus, the flux linking of the windings is also a function of the rotor position. The actual terminal voltage v of the windings can be written in the following form

$$v = \pm \sum_{j=1}^{J} (r_j \cdot i_j) \pm \sum_{j=1}^{J} (\dot{\lambda}_j),$$

where i_j are the currents, r_j are the winding resistances, and λ_j are the flux linkages. The positive directions of the stator currents point out of the synchronous generator terminals.

Thereafter, the two stator electromagnetic fields, both traveling at rotor speed, were identified by decomposing each stator phase current under steady state into two components, one in phase with the electromagnetic field and the other

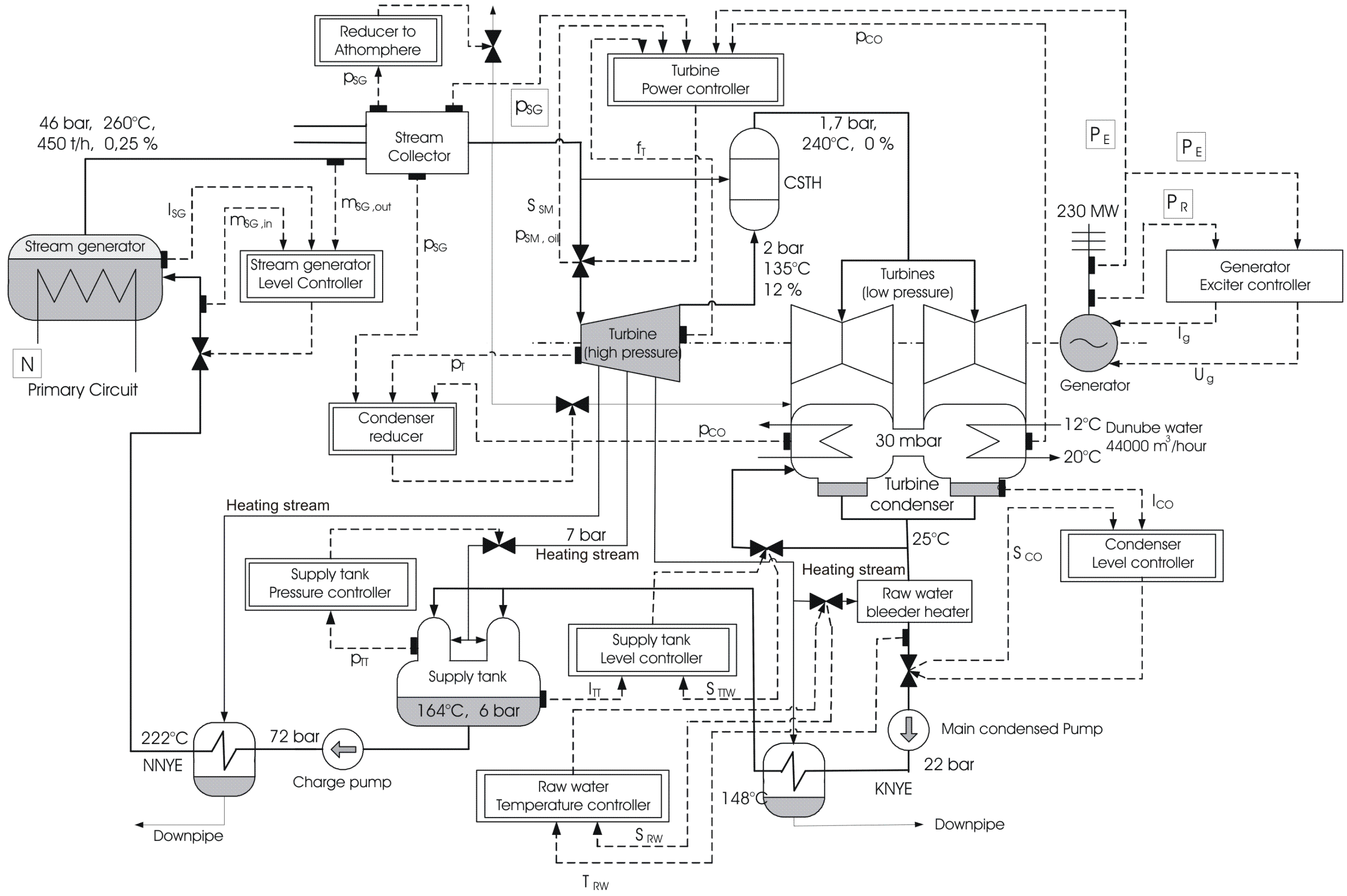

Fig. 2. The schematic figure of the secondary circuit

phase shifted by 90^o. With the aboves, one can construct an airgap field with its maximum aligned to the rotor poles (d axis), while the other is aligned to the q axis (between poles). This method is called the Park's transformation that gives

$$i_d = \frac{2}{3}\left[i_a\cos(\Theta) + i_b\cos\left(\Theta - \frac{2\pi}{3}\right) + i_c\cos\left(\Theta - \frac{4\pi}{3}\right)\right]$$

$$i_q = \frac{2}{3}\left[i_a\sin(\Theta) + i_b\sin\left(\Theta - \frac{2\pi}{3}\right) + i_c\sin\left(\Theta - \frac{4\pi}{3}\right)\right]$$

where i_a, i_b and i_c are the phase currents and Θ [rad] is the angle between the phase current i_a and the current i_d . The Park's transformation uses three variables as d and q axis components (i_d and i_q) and the last one is the stationary current component (i_0), which is proportional to the zero-sequence current. We can get the new current components from the following relationship:

$$i_{0dq} = P \cdot i_{abc}$$

where the current vectors are

$$i_{0dq} = \begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} \quad \text{and} \quad i_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{1}$$

and the Park's transformation matrix is:

$$P = \sqrt{\frac{2}{3}}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ i_a\cos(\Theta) & i_b\cos(\Theta - \frac{2\pi}{3}) & i_c\cos(\Theta - \frac{4\pi}{3}) \\ i_a\sin(\Theta) & i_b\sin(\Theta - \frac{2\pi}{3}) & i_c\sin(\Theta - \frac{4\pi}{3}) \end{bmatrix} \tag{2}$$

All flux components correspond to an electromagnetic field (EMF), the generator EMF is primarily along the rotor q axis. The angle between this EMF and the output voltage is the machine torque angle δ, where the phase a is the reference voltage of the output voltage. The position of the d axis (in radian) is in the form

$$\Theta = \omega_r t + \delta + \pi/2,$$

where ω_r is the rated synchronous angular frequency. Finally, the the following voltage and linkages equations can be written

$$v_{0dq} = P \cdot v_{abc} \quad \text{and} \quad \lambda_{0dq} = P \cdot \lambda_{abc},$$

where the voltage vectors v_{0dg} and v_{abc}, and the linkage flux vectors λ_{0dg} and λ_{abc} are formed similarly to (1).

Because the Park's transformation is unique there exists an inverse of the Park's transformation matrix (2) that is used to obtain:

$$i_{abc} = P^{-1} \cdot i_{0dq}$$

B. The Flux Linkage Equations

The generator consists of six coupled coils referred to with indices a, b, c, F, D and Q, so the linkage equations can be written in the following form:

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

where L_{xy} is the coupling inductance of the coils. It is important to note that the inductances are time varying since Θ is a function of time. The time-varying inductances can be simplified by referring all quantities to a rotor frame of reference through Park's Transformation:

$$\begin{bmatrix} P & 0 \\ 0 & I_3 \end{bmatrix} \begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} P & 0 \\ 0 & I_3 \end{bmatrix} \begin{bmatrix} L_{aa} & L_{aR} \\ L_{Ra} & L_{RR} \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} P^{-1} & 0 \\ 0 & I_3 \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & I_3 \end{bmatrix} \begin{bmatrix} i_{abc} \\ i_{FDQ} \end{bmatrix},$$

where L_{RR} is the rotor-rotor, L_{aa} is the stator-stator, L_{aR} and L_{Ra} are the stator-rotor inductances. The matrix P is the Park's transformation (2) and I_3 is the 3x3 unit matrix. This way, we obtain the following transformed flux linkage equations:

$$\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

Because the variables in their natural units have a few orders of magnitude difference in their values, the equations are normalized using a base value (corresponding to the normal range of the variables). This way all signals are measured in normalized units (p.u.).

C. The State-Space Model

Thereafter the flux linkage equations are extended by the exciting voltage terms as external sources that are considered to be the *manipulated input and disturbance variables* (v_d, v_q and v_F).

With these we can obtain the flux linkage model (7), where l_d, l_F and l_D are the linkage inductances. These equations form the set of state equations of the generator's state-space model. It is important to note that *these state equations are linear and time-invariant*.

The field currents can also be computed from the flux linkage model:

$$\begin{aligned} i_d &= \tfrac{1}{l_d}(\lambda_d - \lambda_D) \\ i_q &= \tfrac{1}{l_q}\lambda_q \\ i_F &= \tfrac{1}{l_F}(\lambda_F - \lambda_D) \end{aligned} \tag{3}$$

Using (3), the output active power equation can be written in the following form:

$$p_{out} = v_d i_d + v_q i_q + v_0 i_0 \tag{4}$$

Assuming steady-state for the stationary components ($v_0 = i_0 = 0$), (4) simplifies to

$$p_{out} = v_d i_d + v_q i_q,$$

and the reactive power is

$$q_{out} = v_d i_q - v_q i_d.$$

Substituting back the flux components, the effective and the reactive power can be written in the following form:

$$p_{out} = v_d \frac{1}{l_d}(\lambda_d - \lambda_D) + v_q \frac{1}{l_q}\lambda_q \tag{5}$$

$$q_{out} = v_d \frac{1}{l_q}\lambda_q - v_q \frac{1}{l_d}(\lambda_d - \lambda_D) \tag{6}$$

Equations (5-6) are the *output equations* of the generator's state-space model. Observe, that these equations *are bi-linear in the state and input variables*.

IV. Simulation Results

The above model has been verified by simulation against engineering intuition using parameter values of a similar generator taken from the literature.

A. Generator Parameters

The parameters of the synchronous generator similar to the one found in NPP were obtained from the literature [1]:

Apparent energy	160 MVA,
$\cos\varphi$	0.85,
Voltage	15 kV,
Current	6158 A,
Frequency	60 Hz,
Excitation current	926 A,
Excitation voltage	375 V.

The stator base quantities, the rated power, output voltage, output current and the angular frequency, that can be freely chosen, are the following

$$\begin{aligned} S_B &= 160 \text{ MVA}/3 = 53.333 \text{ MVA} \\ V_B &= 15 \text{ kV}/\sqrt{3} = 8.66 \text{ kV} \\ I_B &= 6158 \text{ A} \\ \omega_e &= 2\pi f \text{ rad/s} \end{aligned}$$

The time, flux, resistance and inductance quantities are:

$$\begin{aligned} t_B &= 1/(2\pi 60 \text{ Hz}) = 2.626 \text{ ms} \\ \lambda_B &= V_B t_B = 22.972 \text{ Vs} \\ R_B &= V_B/I_B = 1.406\ \Omega \\ L_B &= V_B/(I_B/t_B) = 3.73 \text{ mH} \end{aligned}$$

Finally, the parameters of the synchronous machine and external network in p.u. are:

$L_d = 1.700$	$l_d = 0.150$	$L_{MD} = 0.02838$
$L_q = 1.640$	$l_q = 0.150$	$L_{MQ} = 0.2836$
$L_D = 1.605$	$l_F = 0.101$	$r = 0.001096$
$L_Q = 1.526$	$l_D = -0.055$	$r_F = 0.00074$
$L_{AD} = 1.550$	$l_Q = 0.036$	$r_D = 0.0131$
$L_{AQ} = 1.490$	$r_Q = 0.054$	$R = 100$
$V_\infty = 0.828$		

The steady-state values of the state variables can be obtained from the flux linkage model as state equations (7) by substituting the parameters and by setting the time derivatives equal to zero. This gives the steady-state values of the state variables: $\lambda_d = 1.345$, $\lambda_q = 1.934$, $\lambda_F = 1.633$, $\lambda_D = 1.094$, $\lambda_Q = 0.994$.

$$\begin{bmatrix} \dot{\lambda}_d \\ \dot{\lambda}_F \\ \dot{\lambda}_D \\ \dot{\lambda}_q \\ \dot{\lambda}_Q \end{bmatrix} = \begin{bmatrix} -\frac{r}{l_d}\left(1-\frac{L_{MD}}{l_d}\right) & \frac{r}{l_d}\frac{L_{MD}}{l_F} & \frac{r}{l_d}\frac{L_{MD}}{l_D} & -1 & 0 \\ \frac{r_F}{l_F}\frac{L_{MD}}{l_F} & -\frac{r_F}{l_F}\left(1-\frac{L_{MD}}{l_F}\right) & \frac{r_F}{l_F}\frac{L_{MD}}{l_D} & 0 & 0 \\ \frac{r_D}{l_D}\frac{L_{MD}}{l_D} & \frac{r_D}{l_D}\frac{L_{MD}}{l_F} & -\frac{r_D}{l_D}\left(1-\frac{L_{MD}}{l_D}\right) & 0 & 0 \\ 1 & 0 & 0 & -\frac{r}{l_q}\left(1-\frac{L_{MQ}}{l_d}\right) & \frac{r}{l_q}\frac{L_{MQ}}{l_q} \\ 0 & 0 & 0 & \frac{r_Q}{l_Q}\frac{L_{MQ}}{l_Q} & -\frac{r_Q}{l_Q}\left(1-\frac{L_{MQ}}{l_Q}\right) \end{bmatrix} \begin{bmatrix} \lambda_d \\ \lambda_F \\ \lambda_D \\ \lambda_q \\ \lambda_Q \end{bmatrix} + \begin{bmatrix} -v_d \\ v_F \\ 0 \\ -v_q \\ 0 \end{bmatrix} \tag{7}$$

The A matrix of the state-space model $\dot{x} = Ax + Bu$ can be written in the following form:

$$A = 10^{-3} \begin{bmatrix} -5.927 & 2.049 & 3.743 & -1000 & 0 \\ 1.388 & -5.278 & 3.756 & 0 & 0 \\ 44.72 & 66.282 & -115.33 & 0 & 0 \\ 1000 & 0 & 0 & -5.928 & 5.789 \\ 0 & 0 & 0 & 284.854 & -313.53 \end{bmatrix}$$

The eigenvalues of the state matrix are then:

$$\begin{aligned} \lambda_1 &= -5.686 \cdot 10^{-3} + \mathrm{j}0.999 \\ \lambda_2 &= -5.687 \cdot 10^{-3} - \mathrm{j}0.999 \\ \lambda_3 &= -0.313 \\ \lambda_4 &= -0.117 \\ \lambda_5 &= -3.061 \cdot 10^{-3} \end{aligned}$$

The eigenvalues are negative but their magnitudes are small, thus the system is on the boundary of the stability domain. Furthermore, the two first eigenvalues are complex with a relatively large imaginary part, that indicates the presence of an oscillatory component in the response. This behavior was expected because the direct and the quadrature equivalent circuit of the SG consist only resistances and in inductances, where the resistances give the windings and the iron losses, which are small to decrease the heating of the SG.

B. Changing the Effective Power of the Generator

The dynamic properties of the generator have been investigated in such a way that a single synchronous machine was connected to an infinite bus that models the electrical network (Figure 3). The control schemes of synchronous

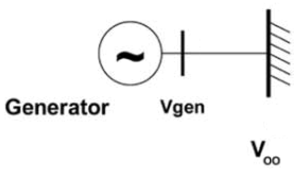

Fig. 3. Synchronous machine connected to an infinite bus.

machines (Figure 4) are commonly based on a reduced-order linearized model and a classical PI controller that ensures stability of the equilibrium point under small perturbations [7]. The controlled output is the effective power (p_{out}) (5), the manipulated input is the exciter voltage (v_F). The proportional parameter of the PI controller is 0.8 and the integrator time is 0.01 in p.u. We have used the same controller in our simulation studies, the controller was implemented in Matlab/Simulink as shown in Figure 4. First we have tested the response of the controlled generator under step-like changes in the setpoint of the effective power. The simulation results are shown in Figure 5, where the currents and the power components (effective and reactive) are shown. It is

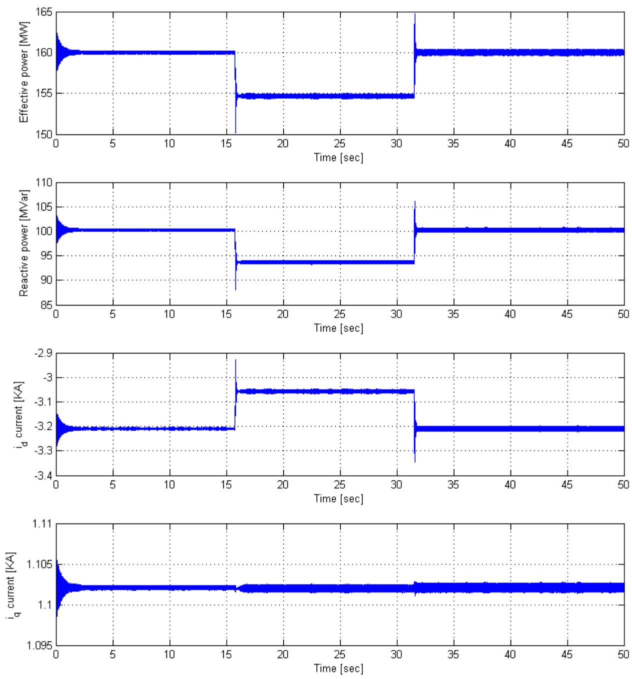

Fig. 5. Changing the effective power of the generator

apparent that both the effective and the reactive power follow the setpoint changes well, and the controlled system is fairly stable.

C. The effect of Disturbances from the Network

The effect of the disturbances from the electrical network is modeled by using a noise from the infinite bus that appears in the voltage variable v_q. We have applied the same step-like changes in the setpoint of the effective power as before.

The simulation results are seen in Figure 6. Here again, both the effective and the reactive power follow the setpoint changes well, and the controlled system is stable. In addition, the controller rejects the disturbances well.

V. Conclusion and Further Work

A simple dynamic model of an industrial size synchronous generator operating in a nuclear power plant is developed in this paper based on first engineering principles. The constructed state-space model consists of an LTI state equation originating from the flux linkage equations, and a bi-linear

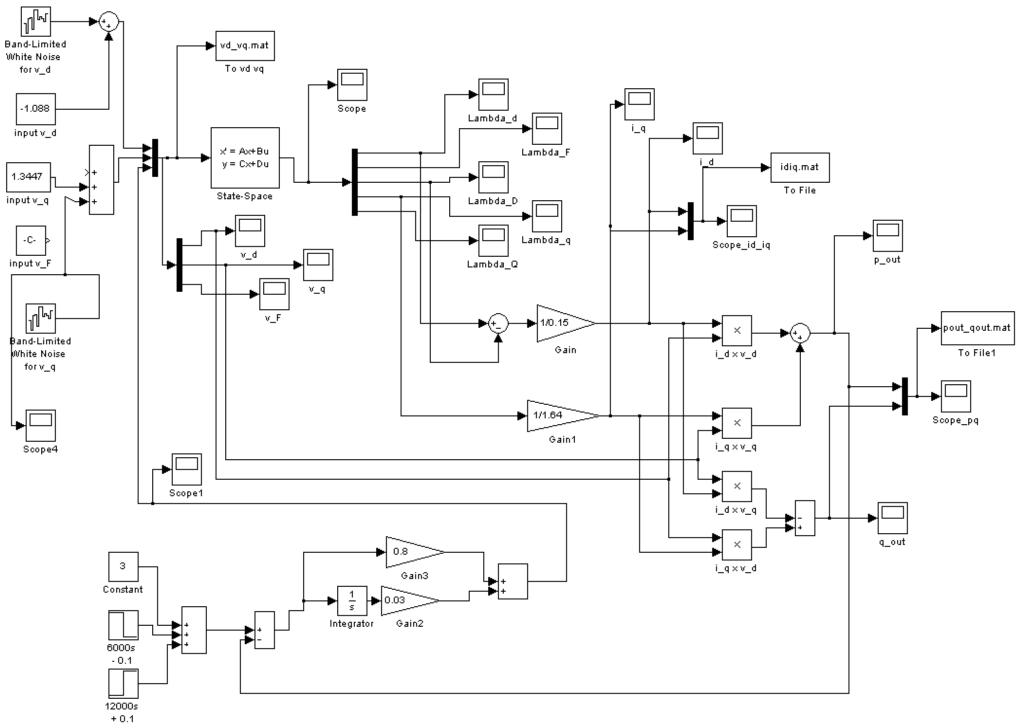

Fig. 4. The controller for the linearized model implemented in Matlab/Simulink

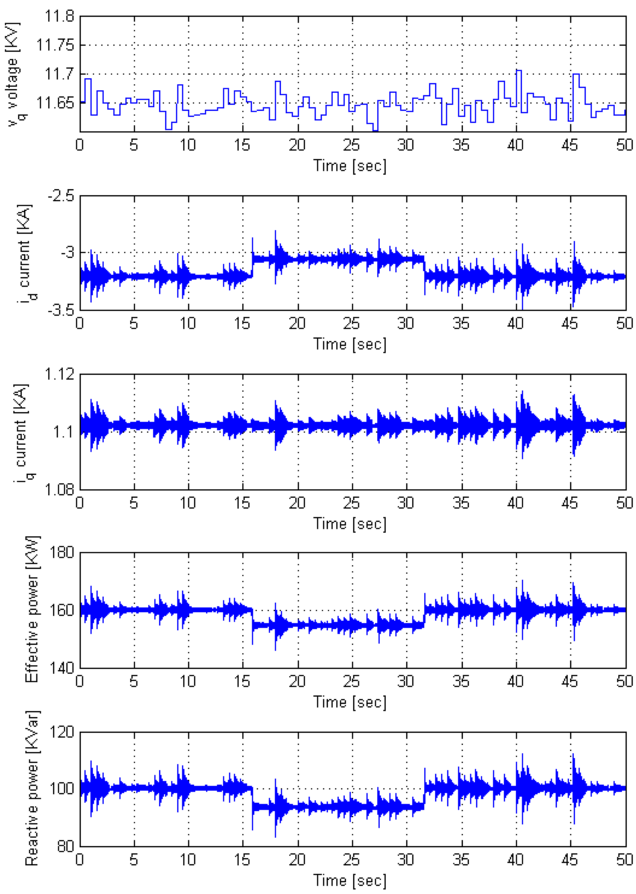

Fig. 6. The effect of the network disturbances

output equation giving the effective and reactive power of the generator.

It has been shown that the model is asymptotically stable with parameters obtained from the literature for a similar generator, but it is very close to the stability boundary.

The effect of load disturbances on the partially controlled generator has been analyzed by simulation by using a traditional PI controller. It has been found that the controlled system is stable and can follow the set-point changes in the effective power well. The disturbance rejection of the controller is also satisfactory.

References

[1] P.M. Anderson and A.A. Fouad: Power-Systems-Control and Stability, *The IOWA State Universty Press, Ames Iowa*, 1977.

[2] R.N. Banavar and U.V. Deshpande: Robust controller design for a nuclear power plant using H-infinity optimization. In *Proc. of the 35th Conference on Decision and Control*, pages 44744479, Kobe, Japan, December 1996.

[3] M.G. Na, D.W. Jung, S.H. Shin, J.W. Jang, K.B. Lee, and Y.J. Lee: A model predictive controller for loadfollowing operation of PWR reactors. *IEEE Transac- tions on Nuclear Science*, **52**:10091020, 2005.

[4] Cs. Fazekas, G. Szederknyi, and K.M. Hangos: A simple dynamic model of the primary circuit in VVER plants for controller design purposes. *Nuclear Engineering and Design*, **237**:10711087, 2007

[5] IAEA. Pressurized water reactor simulator. *IAEA-TCS- 22*, ISSN 1018-5518, IAEA, Vienna, 2003

[6] A. Loukianov, J.M. Canedo, V.I. Utkin and J. Cabrera-Vazquez: Discontinuous controller for power systems: sliding-mode bock control approach. *IEEE Trans. on Industrial Electronics*, **51**:340-353 , 2004

[7] F.P. de Mello and C. Concordia: Concepts of synchronous machine stability as affected by excitation control, *IEEE Trans. Power Application Systems*, **PAS-88**:316-329, 1969.