Functional Adaptive Controller for MIMO Systems with Dynamic Structure of Neural Network

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***Abstract*— A functional adaptive control for nonlinear stochastic Multi-Input Multi-Output (MIMO) systems is presented. A nonlinear system is modelled by a MultiLayer Perceptron (MLP) neural network. Parameters of the model are estimated by the extended Kalman filter (EKF). One of the key problems connected with neural network is selection of its structure. In order to avoid this problem, an on-line algorithm for dynamic structure optimization of the MLP network is proposed. Controller design is based on bicriterial dual approach that uses two separate criteria to introduce opposing aspects between estimation and control; caution and probing. The proposed approach is compared with two adaptive non-dual controllers. The quality of the proposed functional adaptive controller is illustrated in a numerical example.**

I. Introduction

In the last decade, functional adaptive control system design for nonlinear systems has attracted a great deal of attention [1], [2], [3], [4]. The title 'Functional adaptive control' refers to the fact that the type of model uncertainty is connected with functional uncertainty, where the nonlinear functions and parameters of the system are unknown.

Modelling of unknown nonlinear functions that describe system can be approached via functional approximators represented for example by diverse types of neural networks [5] (radial basis function (RBF) or multilayer perceptron (MLP)). One of the most difficult problems connected with neural network is selection of its structure, i.e. the number of hidden layers and the number of neurons in them. Either it is possible to choose a static structure before controller is designed or to use a dynamic structure optimization algorithm. Utilization of dynamic structure allows to set optimal structure of the neural network and so to decrease overall computational demands of the designed controller. In the above mentioned works only a static structure is used.

Typically, functional adaptive control should simultaneously optimize control performance and reduce uncertainty of the system. In contrast to methods that use well known certainty equivalence principle, functional adaptive control system generates action signal representing compromise between control and identification. In addition, it is possible to avoid time consuming off-line system identification. Typically, such methods are either adaptive critic [4] or dual control methods [6].

It should be pointed out that the above mentioned results of functional adaptive control are limited to single-input single-output (SISO) systems. However, many control systems are multivariable [7], [8], [9]. The controller design for multi-input multi-output (MIMO) systems is more difficult and very different from design for SISO systems. The design techniques for SISO systems cannot be simply extended to MIMO systems in general. Hence, problems of representation, identification and control of MIMO systems are more challenging than in case of SISO systems. Even in case of linear MIMO systems with known parameters a task of control design is difficult, especially due to existing coupling between individual input/output channels. The control problem is more complicated in case of nonlinear stochastic systems with uncertainties about nonlinear functions describing the systems. It is a task of the functional adaptive control for MIMO systems. However, this area of functional adaptive control has been addressed marginally up to now.

The first attempt to deal with the theoretical aspects of the representation and control of nonlinear multivariable dynamical systems is presented in [7]. But an intensive off-line training of a neural network is needed. In [10] a comparison of neural networks and gaussian process (GP) models is performed. The certainty equivalence principle is used in control design and so there is no reduction of future uncertainties about a model. Further, the GP techniques requires off-line identification. In [8], the technique of multiple models is used for MIMO control design, but only system with no disturbances is considered. The functional adaptive control for the multivariable discrete-time stochastic systems has not been studied yet.

Hence, main goal of the paper is to design functional adaptive control for non-linear stochastic MIMO systems where unknown functions of a system are modelled by a MLP network, and combine it with a dynamic structure algorithm of neural network.

The paper is organized as follows. In Section 2 the problem of dual stochastic adaptive control for non-linear MIMO systems is formulated. Section 3 concentrates on MIMO system identification by neural networks. The derivation of the bicriterial dual controller is shown in Section 4. In Section 5 the proposed approach is demonstrated in a numerical example.

II. Problem Statement

A dynamical system to be controlled is the nonlinear time-invariant stochastic discrete-time system with m inputs and n outputs given as

$$\mathbf{y}_k = \mathbf{f}(\mathbf{x}_{k-1}) + \mathbf{G}(\mathbf{x}_{k-1})\mathbf{u}_{k-1} + \mathbf{e}_k, \tag{1}$$

where $\mathbf{u}_k = [u_k^{(1)}, \ldots, u_k^{(m)}]^T$ are inputs of the system, $\mathbf{y}_k = [y_k^{(1)}, \ldots, y_k^{(n)}]^T$ denotes outputs of the system, $\mathbf{f}(\mathbf{x}_{k-1}) = [f^{(1)}(\mathbf{x}_{k-1}), \ldots, f^{(n)}(\mathbf{x}_{k-1})]^T$ is the n-dimensional vector of unknown nonlinear functions, further

$$\mathbf{G}(\mathbf{x}_{k-1}) = \begin{bmatrix} g^{(11)}(\mathbf{x}_{k-1}) & \dots & g^{(1m)}(\mathbf{x}_{k-1}) \\ \vdots & \ddots & \vdots \\ g^{(n1)}(\mathbf{x}_{k-1}) & \dots & g^{(nm)}(\mathbf{x}_{k-1}) \end{bmatrix} \qquad (2)$$

is the $n \times m$ matrix of *unknown* nonlinear functions, $\mathbf{x}_{k-1} \triangleq [\mathbf{y}_{k-p}^T, \dots, \mathbf{y}_{k-1}^T, \mathbf{u}_{k-1-s}^T, \dots, \mathbf{u}_{k-2}^T]^T$ is the state of the system and $\mathbf{e}_k = [e_k^{(1)}, \dots, e_k^{(n)}]^T$ is the vector of the additive noises. It is required that the output $\mathbf{y}_k$ follows the chosen reference signal $\mathbf{r}_k = [r_k^{(1)}, \dots, r_k^{(n)}]^T$. It is assumed that the following conditions are satisfied.

Assumption 1: Parameters p and s are known.

Assumption 2: The system has a globally uniformly asymptotically stable zero dynamics [11] and nonlinear functions in the matrix $\mathbf{G}(\mathbf{x}_{k-1})$ are bounded away from zero for all $\mathbf{x}_k$.

Assumption 3: The white noise $\{\mathbf{e}_k\}$ has Gaussian distribution with zero mean and covariance matrix $\boldsymbol{\Xi} = \mathrm{diag}\left(\sigma_e^{(i)}\right)$, where $\sigma_e^{(i)}$ are known variances of the $e_k^{(i)}$ for $i = 1, \dots, n$.

Assumption 4: The relative order of the system is the same for all outputs.

The goal of the control is to design the functional adaptive dual controller for the system (1) in such a way that the output of the system $\mathbf{y}_k$ will follow appropriate reference signal $\mathbf{r}_k$ chosen by designer, in other words to derive the control law by minimization of properly chosen criterion.

Design of the functional adaptive control will be made following way. In terms of solution design is necessary to deal with suitable representation of the MIMO system (1) by neural networks and controller design. The controller design will be based on the bicriterial dual control approach [6]. Attention will be focused on the MLP networks, because they can approximate nonlinear function at the same accuracy as RBF networks with significantly less number of neurons for real time applications. One issue of system identification by MLP networks is estimation of network parameters. In this case, the parameter estimation represents nonlinear optimization problem. It is known that parameter estimation methods are based either on minimization of prediction error [12] or on nonlinear filtering methods [13], [1], [14]. Next key problems joined with neural networks is selection of the neural network structure. In order to avoid this problem, an on-line dynamic structure algorithm of the MLP network is proposed.

III. Model of the MIMO system by neural networks

Firstly, a suitable model of the system (1) have to be specified. MIMO systems are mostly characterized by high dimension of the system state. Hence, MLP network is suitable type of neural network for modelling the MIMO system [7].

An approximation of the system (1) can be made in various ways [7]. Chosen alternative uses two neural networks $\hat{f}^{(i)}$ and $\hat{g}^{(i\cdot)}$ where $i = 1, \dots, n$. Each network $\hat{f}^{(i)}$ has single output and each network $\hat{g}^{(i\cdot)}$ has m outputs. Total number of the networks is $2n$. Although some difficulties are connected with this technique, as design of the $2n$ neural networks and nonlinear estimation of their parameters, this model will be preferred further.

The model of the system is described as

$$\hat{\mathbf{y}}_k = \hat{\mathbf{f}}(\mathbf{x}_{k-1}, \mathbf{w}_k^f, \mathbf{c}_k^f) + \hat{\mathbf{G}}(\mathbf{x}_{k-1}, \mathbf{w}_k^g, \mathbf{c}_k^g)\mathbf{u}_{k-1}, \qquad (3)$$

where the i^{th} output of the model is given as

$$\hat{y}_k^{(i)} = \hat{f}^{(i)}(\cdot) + \sum_{j=1}^{m} \hat{g}^{(ij)}(\cdot) u_{k-1}^{(j)}, \text{ for } i = 1, \dots, n \qquad (4)$$

$$\hat{f}^{(i)}(\mathbf{c}_k^{f_i}, \mathbf{x}_{k-1}^a, \mathbf{w}_k^{f_i}) = (\mathbf{c}_k^{f_i})^T \phi^{f_i}(\mathbf{x}_{k-1}^a, \mathbf{w}_k^{f_i}), \qquad (5)$$

$$\hat{g}^{(ij)}(\mathbf{c}_k^{g_{ij}}, \mathbf{x}_{k-1}^a, \mathbf{w}_k^{g_i}) = (\mathbf{c}_k^{g_{ij}})^T \phi^{g_i}(\mathbf{x}_{k-1}^a, \mathbf{w}_k^{g_i}), \qquad (6)$$

where $\mathbf{x}_{k-1}^a = [\mathbf{x}_{k-1}^T \ , \ 1]^T$ is the state vector augmented by constant bias input, $\mathbf{c}_k^{f_i}$ and $\mathbf{c}_k^{g_{ij}}$ are nf_i, resp. ng_{ij} dimensional vectors of the unknown parameters of the output layer of the network $f^{(i)}$, resp. $g^{(ij)}$, $\mathbf{w}_k^{f_i}$ and $\mathbf{w}_k^{g_i}$ are vectors of the unknown parameters of the hidden layer of the i^{th} network with length $(n+p+1)nf_i$, resp. $(n+p+1)ng_{ij}$. Scalar functions $\phi^{f_i}(\cdot)$ and $\phi^{g_i}(\cdot)$ are sigmoidal activation functions of the neurons in the hidden layers.

Equations (3)–(6) describe the model of the system (1). Before an application of an estimation method for the parameters estimation, a suitable estimation model of the identified system have to be defined. First, all parameters of the model (3) will be included in one parameter vector

$$\boldsymbol{\Theta}_k = \Big[(\mathbf{c}_k^{f_1})^T, (\mathbf{w}_k^{f_1})^T, (\mathbf{c}_k^{g_{11}})^T, \dots, (\mathbf{c}_k^{g_{1m}})^T, (\mathbf{w}_k^{g_1})^T, \dots, (\mathbf{c}_k^{f_n})^T, (\mathbf{w}_k^{f_n})^T, (\mathbf{c}_k^{g_{n1}})^T, \dots, (\mathbf{c}_k^{g_{nm}})^T, (\mathbf{w}_k^{g_n})^T\Big]^T, \qquad (7)$$

where length of the vector $\boldsymbol{\Theta}_k$ is denoted θ_n.

The following two subsections concentrate on searching the optimal structure and the parameter values of the MLP networks representing parameters of the model (3)–(6).

A. Parameter estimation

The parameters of the networks are considered as t-invariant in time

$$\boldsymbol{\Theta}_{k+1} = \boldsymbol{\Theta}_k. \qquad (8)$$

Further, it is assumed that the system (1) can be approximated with sufficient precision by a chosen neural network. Then, it is possible to obtain the measurement equation from (1) by rewriting it as

$$\mathbf{y}_k = \mathbf{h}_k(\boldsymbol{\Theta}_k, \mathbf{x}_{k-1}^a, \mathbf{u}_{k-1}) + \mathbf{e}_k, \qquad (9)$$

where

$$\mathbf{h}_k(\cdot) = \hat{\mathbf{f}}(\mathbf{c}_k^f, \mathbf{x}_{k-1}^a, \mathbf{w}_k^f) + \hat{\mathbf{G}}(\mathbf{c}_k^g, \mathbf{x}_{k-1}^a, \mathbf{w}_k^g)\mathbf{u}_{k-1}, \qquad (10)$$

and

$$\begin{aligned}
\mathbf{w}_k^f &= [(\mathbf{w}_k^{f_1})^T, \ldots, (\mathbf{w}_k^{f_n})^T]^T,\\
\mathbf{w}_k^g &= [(\mathbf{w}_k^{g_1})^T, \ldots, (\mathbf{w}_k^{g_n})^T]^T,\\
\mathbf{c}_k^f &= [(\mathbf{c}_k^{f_1})^T, \ldots, (\mathbf{c}_k^{f_n})^T]^T,\\
\mathbf{c}_k^g &= [(\mathbf{c}_k^{g_{11}})^T, \ldots, (\mathbf{c}_k^{g_{1m}})^T, \ldots, (\mathbf{c}_k^{g_{n1}})^T, \ldots, (\mathbf{c}_k^{g_{nm}})^T]^T.
\end{aligned} \tag{11}$$

Equations (8) and (9) define the estimation model of the system (1). Unfortunately, dependence of $\hat{\mathbf{y}}_k$ on the parameters of the neural network is nonlinear in (3)–(6). Therefore, it is advisable to exploit nonlinear estimation method for finding the unknown parameters. There are many optimization methods developed for training the MLP networks [12], [1]. The methods based on non-linear filtering belong among the most promising and enduring of enhanced training methods because they are practical, computationally moderate and they represent an effective alternative to optimization methods as quasi-Newton, Levenberg-Marquardt, or conjugate gradient techniques [15]. The well-known extended Kalman filter (EKF) represents probably the most attractive approach from non-linear filtering due to the above mentioned characteristic.

The pdf $p(\mathbf{\Theta}_k|\mathbf{I}^k)$ is given as

$$p(\mathbf{\Theta}_k|\mathbf{I}^k) = \mathcal{N}\left\{\mathbf{\Theta}_k : \hat{\mathbf{\Theta}}_k, \mathbf{P}_k\right\}, \tag{12}$$

where

$$\hat{\mathbf{\Theta}}_k = \hat{\mathbf{\Theta}}_{k-1} + \mathbf{K}_k\left[\mathbf{y}_k - \hat{\mathbf{y}}_k\right], \tag{13}$$

$$\mathbf{P}_k = \mathbf{P}_{k-1} - \mathbf{K}_k \nabla_k \mathbf{P}_{k-1}, \tag{14}$$

$$\mathbf{K}_k = \mathbf{P}_{k-1}[\nabla_k]^T\left[\nabla_k \mathbf{P}_{k-1}[\nabla_k]^T + \Xi\right]^{-1}_{,} \tag{15}$$

where ∇_k represents the first derivative of the function $\mathbf{h}_k(\cdot)$ with respect to parameters $\mathbf{\Theta}_k$ and has the following form

$$\nabla_k \triangleq \left.\frac{\partial \hat{\mathbf{h}}(\mathbf{\Theta})}{\partial \mathbf{\Theta}}\right|_{\mathbf{\Theta}=\hat{\mathbf{\Theta}}_k} = \begin{bmatrix} \nabla_k^{(1)} & & & & \mathbf{0} \\ & \ddots & & & \\ & & \nabla_k^{(i)} & & \\ & & & \ddots & \\ \mathbf{0} & & & & \nabla_k^{(n)} \end{bmatrix}, \tag{16}$$

where

$$\nabla_k^{(i)} = \left[\nabla_k^{f_i}, \quad \nabla_k^{g_{i1}} u_{k-1}^{(1)}, \quad \ldots, \quad \nabla_k^{g_{im}} u_{k-1}^{(m)}\right]. \tag{17}$$

Non-diagonal parts in (16) are equal to zero because of the independency of corresponding derivatives on elements of (7). This fact results from the equation of the model (3)–(6) and definition of the vector of the unknown parameters $\mathbf{\Theta}_k$ in (7).

Thus, all information necessary for the control action can be collected on-line at every step k and the neural control scheme with static structure of the neural network can be accomplished. Now, the control scheme will be extended about dynamic structure adaptation process.

B. Dynamic structure optimization

The proposed approach is based on pruning insignificant connections in the network and it is a special case of the algorithm proposed in [16], where more details about the algorithm derivation can be found. Pruning methods are generally based on sufficiently large networks initialization. Then the insignificant connections or neurons are pruned from the networks by setting the corresponding parameter value to zero. The pruning algorithm should be started after stabilization of the mean square error of the predicted output $\varepsilon_k = ||\mathbf{y}_k - \hat{\mathbf{y}}_k||$ which indicates that the network parameters have been set. This is measured by the following difference

$$\Delta_k = \left|\frac{1}{k+1}\sum_{i=0}^{k}\varepsilon_i^2 - \frac{1}{k}\sum_{i=0}^{k-1}\varepsilon_i^2\right|. \tag{18}$$

If the difference Δ_k is less than or equal to a chosen threshold Δ_0 then the pruning process can be started.

The significancy of the s-th parameter of the network can be derived as

$$E_i \approx \frac{\hat{\theta}_s^2}{P_s}, \tag{19}$$

where $\hat{\theta}_s$ and P_s is the s-th term of the vector $\hat{\mathbf{\Theta}}_{k+1}$, resp. the s-th diagonal term of the matrix $\mathbf{P}_{k+1}$.

As the final step, the parameters that should be pruned in the step k must be chosen. For the sake of simplicity index $k+1$ will be omitted in $\hat{\mathbf{\Theta}}_{k+1}$. It is suitable to sort the elements of the vector $\hat{\mathbf{\Theta}}$ according to their significancy as

$$\hat{\mathbf{\Theta}} = [\hat{\theta}_{\pi 1}, \hat{\theta}_{\pi 2}, \ldots, \hat{\theta}_{\pi\theta n}], \tag{20}$$

where $\hat{\theta}_{\pi 1}$ is parameter with the lowest significancy E_i.

A criterion for pruning connections from the network is set as

$$T = \frac{1}{k+1}(\hat{\mathbf{\Theta}}_{[1,\pi N]} - \hat{\mathbf{\Theta}})^T \mathbf{P}^{-1}(\hat{\mathbf{\Theta}}_{[1,\pi N]} - \hat{\mathbf{\Theta}}), \quad \pi N = 1, \ldots, \theta n \tag{21}$$

where $\hat{\mathbf{\Theta}}_{[1,\pi N]} = [0, \ldots, 0, \theta_{\pi N+1}, \theta_{\pi N+2}, \ldots, \theta_{\pi\theta n}]$. If the value T will be less than the chosen threshold T_0, then the connections will be pruned. The strategy is based on finding such sequence of the parameters that their pruning causes only a minor error in the model. For that reason, a sequential computation of the criterion (21) from the parameter with the lowest significancy to the most significant is performed until the condition $T < T_0$ is satisfied.

Now, it is possible to obtain both the estimate and the covariance matrix of the parameters of the system at every step of estimation algorithm which are necessary for computation of a control action $\mathbf{u}_k$. Due to chosen neural network structure, the covariance matrix $\mathbf{P}_{k+1}$ can by written in a block diagonal form as

$$\mathbf{P}_{k+1} = \begin{bmatrix} \mathbf{P}_{k+1}^{(1,1)} & & & & \mathbf{0} \\ & \ddots & & & \\ & & \mathbf{P}_{k+1}^{(i,i)} & & \\ & & & \ddots & \\ \mathbf{0} & & & & \mathbf{P}_{k+1}^{(n,n)} \end{bmatrix}, \tag{22}$$

where

$$\mathbf{P}_{k+1}^{(i,i)}=\begin{bmatrix}\mathbf{P}_{k+1}^{f_if_i} & \mathbf{P}_{k+1}^{f_ig_{i1}} & \dots & \mathbf{P}_{k+1}^{f_ig_{im}}\\ \mathbf{P}_{k+1}^{g_{i1}f_i} & \mathbf{P}_{k+1}^{g_{i1}g_{i1}} & \dots & \mathbf{P}_{k+1}^{g_{i1}g_{im}}\\ \vdots & \vdots & \vdots & \vdots\\ \mathbf{P}_{k+1}^{g_{im}f_i} & \mathbf{P}_{k+1}^{g_{im}g_{i1}} & \dots & \mathbf{P}_{k+1}^{g_{im}g_{im}}\end{bmatrix} \quad (23)$$

is an individual sub-matrix having dimensions given by numbers of parameters relevant to the neural networks $\hat{f}^{(i)}$, $\hat{g}^{(ij)}$.

IV. Bicriterial dual control design

In this section functional adaptive control for the system (1) will be designed using the idea of bicriterial dual control (BDC). It can be mentioned that basic idea of bicriterial approach is based on subsequent minimization of two critera. These criteria represent two opposite goals of the dual control: identification and control.

The first criterion evaluating the control quality is described as

$$J_k^c = E\big\{(\mathbf{y}_{k+1}-\mathbf{r}_{k+1})^T\mathbf{Q}_{k+1}(\mathbf{y}_{k+1}-\mathbf{r}_{k+1})+ \mathbf{u}_k^T\mathbf{S}_{k+1}\mathbf{u}_k)|\mathbf{I}_k\big\}, \quad (24)$$

where $E\{\cdot|\cdot\}$ is conditional expectation operator, $\mathbf{Q}_{k+1}$ is $n\times n$ positive semidefinite matrix, $\mathbf{S}_{k+1}$ is $m\times m$ positive definite matrix and $\mathbf{I}_k$ describes available information segment until time k.

Remark 4.1: The arguments of nonlinear functions $\mathbf{f}$, $\mathbf{G}$, $\hat{\mathbf{f}}$ and $\hat{\mathbf{G}}$ will be omitted in the following derivation for abbreviation of notation.

By substituting (1) into (24) the criterion J_k^c can be rewritten as

$$J_k^c = E\big\{(\mathbf{f}+\mathbf{G}\mathbf{u}_k+\mathbf{e}_k-\mathbf{r}_{k+1})^T\mathbf{Q}_{k+1}\times (\mathbf{f}+\mathbf{G}\mathbf{u}_k+\mathbf{e}_k-\mathbf{r}_{k+1})+\mathbf{u}_k^T\mathbf{S}_{k+1}\mathbf{u}_k|\mathbf{I}_k\big\}, \quad (25)$$

where the functions $\mathbf{f}$, $\mathbf{G}$ should be regarded as random variables. Subsequent multiply and partially application of mean operator over information segment $\mathbf{I}_k$ one can obtain

$$\begin{aligned}J_k^c =& E\{\mathbf{f}^T\mathbf{Q}_{k+1}\mathbf{G}\}\mathbf{u}_k+\mathbf{u}_k^TE\{\mathbf{G}^T\mathbf{Q}_{k+1}\mathbf{f}\}+\\ &\mathbf{u}_k^TE\{\mathbf{G}^T\mathbf{Q}_{k+1}\mathbf{G}\}\mathbf{u}_k-\mathbf{r}_{k+1}^TE\{\mathbf{Q}_{k+1}\mathbf{G}\}\mathbf{u}_k-\\ &\mathbf{u}_k^TE\{\mathbf{G}^T\mathbf{Q}_{k+1}\}\mathbf{r}_{k+1}^T+\mathbf{u}_k^T\mathbf{S}_{k+1}\mathbf{u}_k+c,\end{aligned} \quad (26)$$

where c represents all terms of J_k^c that are independent of $\mathbf{u}_k$. They can not influence value of the criterion and need not be considered further.

Now, it is possible to determine control action $\mathbf{u}_k^c$ as extreme of criterion J_k^c

$$\frac{\partial J_k^c}{\partial \mathbf{u}_k} = E\{\mathbf{G}^T\mathbf{Q}_{k+1}\mathbf{f}\}+E\{\mathbf{G}^T\mathbf{Q}_{k+1}\mathbf{G}\}\mathbf{u}_k- E\{\mathbf{G}^T\mathbf{Q}_{k+1}\}\mathbf{r}_{k+1}+\mathbf{S}_{k+1}\mathbf{u}_k=\mathbf{0}. \quad (27)$$

In the remaining terms in (27) mean operator can be applied by using well known relation $E\{\mathbf{a}^T\mathbf{Q}_{k+1}\mathbf{a}\}=\hat{\mathbf{a}}^T\mathbf{Q}_{k+1}\hat{\mathbf{a}}+E\{[\mathbf{a}-\hat{\mathbf{a}}]^T\mathbf{Q}_{k+1}[\mathbf{a}-\hat{\mathbf{a}}]\}$. Then, the control action $\mathbf{u}_k^c$ can be written as

$$\mathbf{u}_k^c = [\mathbf{S}_{k+1}+\hat{\mathbf{G}}^T\mathbf{Q}_{k+1}\hat{\mathbf{G}}+\boldsymbol{\nu}_{k+1}^{GG}]^{-1}\times [\hat{\mathbf{G}}^T\mathbf{Q}_{k+1}\mathbf{r}_{k+1}-\hat{\mathbf{G}}^T\mathbf{Q}_{k+1}\hat{\mathbf{f}}-\boldsymbol{\nu}_{k+1}^{GF}], \quad (28)$$

where

$$\begin{aligned}&\boldsymbol{\nu}_{k+1}^{GG}=\mathbf{Q}_{k+1}\boldsymbol{\nabla}_{k+1}^{G}\mathbf{P}_{k+1}^{G}(\boldsymbol{\nabla}_{k+1}^{G})^T \text{ is } m\times m \text{ matrix}\\ &\boldsymbol{\nu}_{k+1}^{GF}=\mathbf{Q}_{k+1}\boldsymbol{\nabla}_{k+1}^{G}\mathbf{P}_{k+1}^{GF}(\boldsymbol{\nabla}_{k+1}^{F})^T \text{ is vector with length } m.\end{aligned} \quad (29)$$

Matrices $\mathbf{P}_{k+1}^{G}$ and $\mathbf{P}_{k+1}^{GF}$ can be formed from the elements of the matrix $\mathbf{P}_{k+1}$ described by (22) and have the form given by (30) and $\boldsymbol{\nabla}_{k+1}^{G}$, resp. $\boldsymbol{\nabla}_{k+1}^{F}$ can be obtained from (16) as

$$\begin{aligned}\boldsymbol{\nabla}_{k+1}^{F}&=\begin{bmatrix}\boldsymbol{\nabla}_{k+1}^{f_1}, & \dots, & \boldsymbol{\nabla}_{k+1}^{f_m}\end{bmatrix}^T,\\ \boldsymbol{\nabla}_{k+1}^{G}&=\begin{bmatrix}\boldsymbol{\nabla}_{k+1}^{g_{11}} \ \dots \ \boldsymbol{\nabla}_{k+1}^{g_{1m}} & & \mathbf{0}\\ & \ddots & \\ \mathbf{0} & & \boldsymbol{\nabla}_{k+1}^{g_{m1}} \ \dots \ \boldsymbol{\nabla}_{k+1}^{g_{mm}}\end{bmatrix}.\end{aligned} \quad (31)$$

It can be noted that (28) respects uncertainties in knowledge of the unknown functions and it is equal to cautious control as one of integral component of dual control.

The second component of control law should evaluate estimation quality and it is given by criterion

$$J_k^a = -E\left\{(\mathbf{y}_{k+1}-\hat{\mathbf{y}}_{k+1})^T\mathbf{W}_{k+1}(\mathbf{y}_{k+1}-\hat{\mathbf{y}}_{k+1})|\mathbf{I}_k\right\}, \quad (32)$$

where $\mathbf{W}_{k+1}$ is $n\times n$ positive semidefinite matrix. By substituting (1) and (3) in (32), the following relation can be obtained

$$\begin{aligned}J_k^a =& -E\big\{(\mathbf{f}+\mathbf{G}\mathbf{u}_k+\mathbf{e}_k)^T\mathbf{W}_{k+1}(\mathbf{f}+\mathbf{G}\mathbf{u}_k+\mathbf{e}_k)-\\ &(\mathbf{f}+\mathbf{G}\mathbf{u}_k+\mathbf{e}_k)^T\mathbf{W}_{k+1}(\hat{\mathbf{f}}+\hat{\mathbf{G}}\mathbf{u}_k)-\\ &(\hat{\mathbf{f}}+\hat{\mathbf{G}}\mathbf{u}_k)^T\mathbf{W}_{k+1}(\mathbf{f}+\mathbf{G}\mathbf{u}_k+\mathbf{e}_k)+\\ &(\hat{\mathbf{f}}+\hat{\mathbf{G}}\mathbf{u})^T\mathbf{W}_{k+1}(\hat{\mathbf{f}}+\hat{\mathbf{G}}\mathbf{u}_k)|\mathbf{I}_k\big\}.\end{aligned} \quad (33)$$

After multiplying and omitting the terms independent of $\mathbf{u}_k$ a suitable form for optimization is

$$\begin{aligned}\bar{J}_k^a =& -E\big\{\mathbf{u}_k^T\mathbf{G}^T\mathbf{W}_{k+1}\mathbf{f}+\mathbf{u}_k^T\mathbf{G}^T\mathbf{W}_{k+1}\mathbf{G}\mathbf{u}_k+\mathbf{f}^T\mathbf{W}_{k+1}\times\\ &\mathbf{G}\mathbf{u}_k-\mathbf{f}^T\mathbf{W}_{k+1}\times\hat{\mathbf{G}}\mathbf{u}_k-\mathbf{u}_k^T\mathbf{G}^T\mathbf{W}_{k+1}\hat{\mathbf{G}}\mathbf{u}_k-\\ &\mathbf{u}_k^T\mathbf{G}^T\mathbf{W}_{k+1}\hat{\mathbf{f}}-\hat{\mathbf{f}}^T\mathbf{W}_{k+1}\mathbf{G}\mathbf{u}_k-\mathbf{u}_k^T\hat{\mathbf{G}}^T\mathbf{W}_{k+1}\mathbf{f}-\\ &\mathbf{u}_k^T\hat{\mathbf{G}}^T\mathbf{W}_{k+1}\mathbf{G}\mathbf{u}_k+\hat{\mathbf{f}}^T\mathbf{W}_{k+1}\hat{\mathbf{G}}\mathbf{u}_k+\\ &\mathbf{u}_k^T\hat{\mathbf{G}}^T\mathbf{W}_{k+1}\mathbf{f}+\mathbf{u}_k^T\hat{\mathbf{G}}^T\mathbf{W}_{k+1}\hat{\mathbf{G}}\mathbf{u}_k|\mathbf{I}_k\big\}.\end{aligned} \quad (34)$$

The bicriterial control $\mathbf{u}_k$ is then given as

$$\mathbf{u}_k = \underset{\mathbf{u_k}\in\Omega_\mathrm{k}}{\operatorname{argmin}}\ \bar{J}_k^a. \quad (35)$$

Minimization of $\bar{J}_k^a$ is performed over the region Ω_k which is specified by $\mathbf{u}_k^c$ and its surrounding symmetrically distributed around the cautious control as $\Omega_k=[\mathbf{u}_k^c-\boldsymbol{\delta}_k,\mathbf{u}_k^c+\boldsymbol{\delta}_k]$, where $\boldsymbol{\delta}_k=[\delta_k^{(1)},\dots,\delta_k^{(m)}]^T$. The choice of the parameter $\boldsymbol{\delta}_k$ stems from reasoning that it is necessary to enrich the cautious control with probing proportional to uncertainty of the unknown functions $\mathbf{f}$, $\mathbf{G}$ in the controlled system (1). A common choice [6] for $\boldsymbol{\delta}_k$ is

$$\boldsymbol{\delta}_k=\boldsymbol{\eta}\,\mathrm{tr}(\mathbf{P}_{k+1}), \qquad \eta^{(i)}>0 \ \ \forall\ i, \quad (36)$$

$$\mathbf{P}_{k+1}^{GF} = \begin{bmatrix} \mathbf{P}_{k+1}^{f_1 g_{11}} & \dots & \mathbf{P}_{k+1}^{f_1 g_{1m}} \\ \vdots & \ddots & \vdots \\ \mathbf{P}_{k+1}^{f_m g_{m1}} & \dots & \mathbf{P}_{k+1}^{f_m g_{mm}} \end{bmatrix}, \qquad \mathbf{P}_{k+1}^{G} = \begin{bmatrix} \begin{matrix} \mathbf{P}_{k+1}^{g_{11} g_{11}} & \dots & \mathbf{P}_{k+1}^{g_{11} g_{1m}} \\ \vdots & \vdots & \vdots \\ \mathbf{P}_{k+1}^{g_{11} g_{12}} & \dots & \mathbf{P}_{k+1}^{g_{1m} g_{1m}} \end{matrix} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \begin{matrix} \mathbf{P}_{k+1}^{g_{m1} g_{m1}} & \dots & \mathbf{P}_{k+1}^{g_{m1} g_{mm}} \\ \vdots & \vdots & \vdots \\ \mathbf{P}_{k+1}^{g_{mm} g_{m1}} & \dots & \mathbf{P}_{k+1}^{g_{mm} g_{mm}} \end{matrix} \end{bmatrix} \quad (30)$$

where $\boldsymbol{\eta}$ is m dimensional vector, that provides the amplitude of the probing signal and the matrix $\mathbf{P}_{k+1}$ describes a degree of uncertainty of the parameter estimate conditioned by $\mathbf{I}_k$ and can be obtained using a nonlinear estimation method.

Relation (35) can be rearanged and written as

$$\mathbf{u}_k = \mathbf{u}_k^c + \boldsymbol{\delta}_k \operatorname{sign}\left[\bar{J}_k^a(\mathbf{u}_k^c - \boldsymbol{\delta}_{\boldsymbol{k}}) - \bar{J}_k^a(\mathbf{u}_k^c + \boldsymbol{\delta}_{\boldsymbol{k}})\right]. \quad (37)$$

Now, evaluation of the term in the square brackets remains to be performed. This term can be obtained by substituting $(\mathbf{u}_k^c + \boldsymbol{\delta}_k)$, resp. $(\mathbf{u}_k^c - \boldsymbol{\delta}_k)$ for $\mathbf{u}_k$ in (34). Using elementary adjustments, the following form can be obtained

$$\begin{aligned}\bar{J}_k^a(\mathbf{u}_k^c - \boldsymbol{\delta}_k) - \bar{J}_k^a(\mathbf{u}_k^c + \boldsymbol{\delta}_k) = 4\boldsymbol{\delta}_k^T E\{(\mathbf{G} - \hat{\mathbf{G}})^T \mathbf{W}_{k+1} \times \\ (\mathbf{f} - \hat{\mathbf{f}}) + (\mathbf{G} - \hat{\mathbf{G}})^T \mathbf{W}_{k+1} (\mathbf{G} - \hat{\mathbf{G}}) \mathbf{u}_k^c | \mathbf{I}_k\}.\end{aligned} \quad (38)$$

Now, it is possible to reuse the introduced relations (29). By application of the mean operator and using assumption $\mathbf{Q}_{k+1} = \mathbf{W}_{k+1}$, the equation (38) has form

$$\bar{J}_k^a(\mathbf{u}_k^c - \boldsymbol{\delta}_k) - \bar{J}_k^a(\mathbf{u}_k^c + \boldsymbol{\delta}_k) = 4\boldsymbol{\delta}_k^T(\boldsymbol{\nu}_{k+1}^{GF} + \boldsymbol{\nu}_{k+1}^{GG}\mathbf{u}_k^c). \quad (39)$$

Final equation of functional adaptive controller for MIMO system based on bicriterial approach can be obtained as combination of (27), (37) and (39) and it possible to write

$$\mathbf{u}_k = \mathbf{u}_k^c + \boldsymbol{\delta}_k \operatorname{sign}\left[\boldsymbol{\delta}_k^T(\boldsymbol{\nu}_{k+1}^{GF} + \boldsymbol{\nu}_{k+1}^{GG}\mathbf{u}_k^c)\right]. \quad (40)$$

V. Numerical example

The discrete-time nonlinear stochastic system with two inputs and two outputs described by the following equations is considered

$$\begin{aligned} y_k^{(1)} =& \frac{0.7 y_{k-1}^{(1)} y_{k-2}^{(1)}}{1 + (y_{k-1}^{(1)})^2 + (y_{k-2}^{(2)})^2} + \frac{0.1 u_{k-1}^{(2)}}{1 + 3(y_{k-2}^{(1)})^2 + (y_{k-1}^{(2)})^2} + \\ &+ u_{k-1}^{(1)} + 0.25 u_{k-2}^{(1)} + 0.5 u_{k-2}^{(2)} + e_k^{(1)}, \\ y_k^{(2)} =& \frac{0.5 y_{k-1}^{(2)} \sin y_{k-2}^{(2)}}{1 + (y_{k-1}^{(2)})^2 + (y_{k-2}^{(1)})^2} + 0.5 u_{k-2}^{(2)} + 0.3 u_{k-2}^{(1)} + \\ &+ u_{k-1}^{(2)} \left(0.1 u_{k-2}^{(2)} - 1.5\right) + e_k^{(2)}, \end{aligned}$$

where $\mathbf{x}_{k-1} = [\mathbf{y}_{k-1}, \mathbf{y}_{k-2}, \mathbf{u}_{k-2}]$ is the state of the system, $\{e^{(1)}\}$, $\{e^{(2)}\}$ are mutually independent Gaussian noises with zero means and variances $(\sigma_e^{(1)})^2 = (\sigma_e^{(2)})^2 = 0.005$. Reference signals are chosen as

$$\begin{aligned} r_k^{(1)} =& 0.55 \sin \frac{2\pi k}{30} + 0.55 \sin \frac{2\pi k}{20}, \\ r_k^{(2)} =& 0.75 \sin \frac{2\pi k}{50} + 0.75 \sin \frac{2\pi k}{10}. \end{aligned} \quad (41)$$

Initial values of the inputs and the outputs are zero. As it was mentioned in Section 2, each of the nonlinear functions $f^{(i)}$, $g^{(i\cdot)}$ for $i = 1, \dots, n$ is modelled by individual neural network. Model of the system is composed of four neural networks. Each of the neural networks is perceptron neural network with one hidden layer containing 20 neurons for each function $\hat{f}(\cdot)$ and 15 neurons for each function $\hat{g}(\cdot)$. The initial parameters are generated from uniform distribution on the interval $< -0.1; 0.1 >$ and covariance matrix is $\mathbf{P}_0 = 10\mathbf{I}$. Finally, parameters of the BDC are chosen as follows: $\mathbf{W}_{k+1} = \mathbf{Q}_{k+1} = \mathbf{I}$, $\mathbf{S}_{k+1} = 0.01\mathbf{I}$ and $\boldsymbol{\eta}^T = [0.00002 \ 0.00004]$. Parameters of the pruning algorithm are chosen as $\Delta_0 = 0.005$ and $T_0 = 0.00001$.

Influence of dynamic structure optimization on the control quality and time demands is shown in Table 1. The BDC with dynamic structure optimization is compared with BDC without utilization of the dynamic structure algorithm. Criterion for comparison is set as the mean of sums of square errors of the reference $\mathbf{r}_k$ and the system output $\mathbf{y}_k$ over 100 trials and 300 steps per trial:

$$\hat{V} = \frac{1}{100} \sum_{i=1}^{2} \sum_{j=1}^{100} \sum_{k=1}^{300} (y_{kj}^{(i)} - r_{kj}^{(i)})^2. \quad (42)$$

Utilization of the dynamic structure algorithm brings comparable control quality and significantly lower computational demands in comparison with the static structure. The number of the parameters decreases by 80 % and computational demands decrease by 20 % in the case of dynamic structure optimization. It should be noted that the size of reduction grows with system state dimension and control horizon in general.

Results of the simulation are illustrated in Figures 1 and 2. In Figure 1 the tracking of the chosen reference signals for both outputs of the system $y_k^{(1)}$ and $y_k^{(2)}$ is shown. It is clear that quick adaptation of the parameters of the model occurs up to the 100th step. Very good control quality is achieved after this adaptation period. The goal of the control, i.e. the

TABLE I

INFLUENCE OF DYNAMIC STRUCTURE ON QUALITY OF THE CONTROL SYSTEM AND TIME DEMANDS.

	$\hat{V}$	$\mathrm{cov}(\hat{V})$	$n\theta$	time [s]
BDC stat	27.8	15.8	590	57.2
BDC dynam	26.5	18.2	112	45.5

tracking of the chosen reference signals $r_k^{(1)}$ and $r_k^{(2)}$, is fulfilled. Progress of number of the network parameters in time is illustrated in Fig. 2. Reduction in number of neurons in the neural network is obvious.

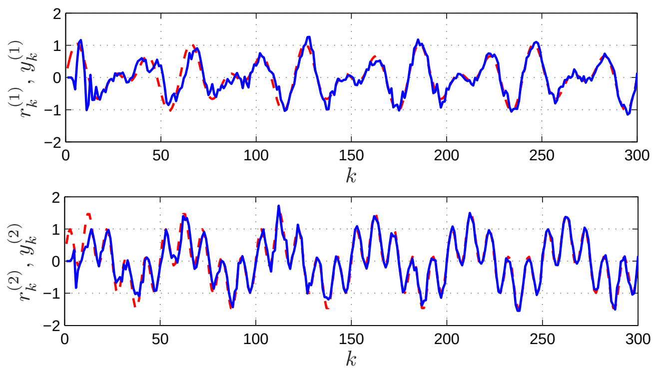

Fig. 1. Typical outputs of the system controlled by bicriterial dual controller with dynamic structure (blue line) and following chosen reference signals (red line).

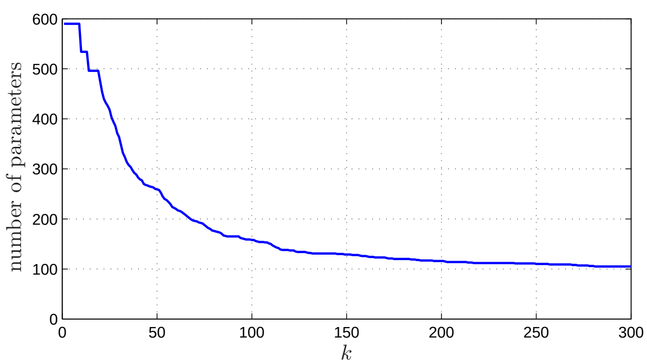

Fig. 2. Progress of the neural network parameters in time.

VI. CONCLUSIONS

The bicriterial dual controller for non-linear stochastic MIMO systems with dynamic structure of perceptron neural network was presented. The model of the system is given by the multi-layer perceptron network. The extended Kalman filter was applied for the on-line parameter estimation of the derived estimation model. In order to avoid the problem with choice of the neural network structure, an on-line dynamic structure optimization algorithm of the network was utilized. Then the bicriterial approach to dual control design was used. The proposed dual adaptive controller with dynamic structure has lower computational demands and comparable control quality in comparison with controller that utilizes static structure of the neural network.

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