

Stability Analysis of Mean-CVaR Investment Model with Transaction Costs and Integer Allocations

Martin Branda

Abstract—Since Markowitz published his pioneer work [15], the performance of a portfolio of assets has been measured by its expected return and risk. However, his model had some drawbacks. Transaction costs, indivisible assets and asymmetrical quantitation of risk were not included into his investment model.

We deal with optimal investment problem with integer allocations, transactions costs and Conditional Value at Risk measure. The underlying distribution is only estimated. Hence, stability analysis with respect to some changes of the distribution is necessary. We propose contamination techniques, which enable us to quantify the change in optimal value, if the underlying distribution is contaminated by another distribution. They provide a way how to construct contamination bounds for optimal value, which quantify the effect of considered change in probability distribution.

We apply introduced investment model to real data of 30 Czech investment funds. We study in-sample and out-of-sample performance of portfolios with different risk aversions and apply contamination techniques to study the behaviour of the risk before and during distress.

I. INTRODUCTION

A. Mean-risk models and multiobjective optimization

Due to the pioneer work of Harry Markowitz [15], the performance of a portfolio of assets is measured not only by its expected return but also by its risk. Searching the optimal portfolio leads to multiobjective (biobjective) optimization problem where we maximize the expected return (minimize expected loss) and minimize the risk at the same time. Markowitz used variance of the portfolio as the risk measure. Many risk measures have been introduced and studied since then: Mean absolute deviation (MAD) [13], Value at Risk (VaR), Conditional Value at Risk (CVaR) [17], drawdown measures [18] and many others. In past years, VaR became very popular risk measure and is nowadays widely used in practice even though it is not adequate risk measure, cf. [22]. It lacks subadditivity which is crucial for correct diversification of our investments. Conditional Value at Risk (CVaR) is often proposed as an alternative for VaR. CVaR is usually defined as the conditional mean of losses on condition that we are beyond VaR. Since this definition leads to some inaccuracies, more accurate definition is needed for general loss distributions, see [17]. If CVaR is defined correctly, it fulfils axioms of coherent risk measures [5] which are accepted by theorists as well as by practitioners.

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M. Branda is with Charles University in Prague, Faculty of Mathematics and Physics, Department of Probability and Mathematical Statistics, Sokolovska 83, Prague 8, 18675, The Czech Republic, branda@karlin.mff.cuni.cz

Minimizing risk $\mathcal{R}(x)$ and maximizing expected return $\mathcal{E}(x)$ at the same time under some common constraints $X \subset \mathbb{R}^n$ on portfolio composition leads to multiobjective (biobjective) optimization problems. We are looking for efficient solutions, i.e. solutions $\hat{x} \in X$ such that there is no element $x \in X$ with $\mathcal{R}(x) \leq \mathcal{R}(\hat{x})$ and $\mathcal{E}(x) \geq \mathcal{E}(\hat{x})$ with at least one strict inequality. There are two main approaches for solving such problems, both leading to single objective problems and under mild condition to efficient solutions, see [16]: aggregate function (weighted sum) approach

$$\min_{x \in X} \left[-(1 - \rho)\mathcal{E}(x) + \rho\mathcal{R}(x) \right]$$

for some $\rho \in (0, 1)$, and ε -constraint approach

$$\begin{aligned} \min_{x \in X} \mathcal{R}(x) \\ \mathcal{E}(x) \geq r^{\min} \end{aligned}$$

with r^{\min} such that $\{x \in X : \mathcal{E}(x) \geq r^{\min}\}$ is nonempty.

In the case of multiobjective linear programming, there is even known relation between both approaches. Using linear programming duality we are able to obtain explicit relation between weighting coefficients and ε -bounds, i.e. between ρ and r^{\min} , cf. [12].

For Markowitz model, both approaches lead to quadratic programming problems. New risk measures are proposed in order to lead to linear programming formulation for discrete random variables which is easier to solve than quadratic problems.

However, introducing integer restriction to some variables destroys convexity of the underlying problem and makes it difficult to solve. The aggregate function approach still leads to efficient solutions, but we are not able to obtain all them. Hence, we only approximate the efficient frontier which contains mean returns and corresponding risks for efficient solutions. Why integer variables are necessary in real financial applications? They help us model indivisible assets (we can buy only integral number of assets), transaction costs, cardinality constraints (restrictions on maximal number of kinds of assets), logical relations (if you buy certain asset, you must not buy other) etc.

B. Stochastic programming

Mean-risk investment models can be seen as a special class of stochastic programming problems. Stochastic programming solves many real-life problems where optimization and randomness appear. Stochastic programming problems arise in economy, finance, industry, agriculture and logistics, cf. [23].

Incorporating integer variables into optimization problems leads in many cases to more realistic models, however, the resulting problems are much more theoretically and computationally demanding. There was a large development in stochastic integer programming in theory and algorithms during last decade [20], [21].

Successful application of stochastic programming problems requires perfect knowledge of underlying probability distribution of random parts. However, the distribution is usually estimated or approximated. Hence, stability analysis with respect to changes of the distribution is necessary, cf. [10]. There are approaches that enable us to estimate the change in the optimal value of our problem if some change in the underlying distribution occurs. Popular theoretical approach is based on probability metrics, cf. [19], which enables us to bound distance between optimal values of our optimization problem with different underlying measures using appropriate probability metrics. However, it can be difficult to compute the metrics, especially in integer stochastic programming where very complicated metrics appear. On the other hand, contamination techniques are more computationally tractable than the approach based on probability metrics, however less general. In this paper we investigate the application of contamination techniques in investment problems with real features, where integer allocations, transactions costs and Conditional Value at Risk measure are considered.

The paper is organized as follows. In Section II, contamination techniques for general stochastic optimization problems are reviewed. In Section III, one period mean-risk model with real features is introduced. CVaR is used to measure the risk of portfolios, integer restrictions and transaction costs are involved. Section IV contains numerical study where the introduced mean-risk model is applied to real portfolio problem with Czech investment funds.

II. CONTAMINATION TECHNIQUES

In this chapter we review briefly general concept of Gateaux directional differentiability and its application in contamination techniques for general stochastic programming problems. We may refer to [8], [9] for introduction and main theoretical results, to [7] for applications in stochastic integer programming and to [11] for risk modelling with Value at Risk, Conditional Value at Risk. This article combines approaches from [7], [11].

In general, we may consider the following stochastic optimization problem

$$\inf_{x \in X} g(x, P), \quad (1)$$

where X is a closed subset of \mathbb{R}^n , the underlying probability measure P belongs to a general class of Borel probability measures \mathcal{P} with support $\Xi \subseteq \mathbb{R}^m$, and g is an objective function from $\mathbb{R}^n \times \mathcal{P}$ to extended real numbers. To apply the contamination techniques, the objective function is assumed to be concave or even linear in P . The latter is true, e.g. for objective functions of expectation type.

Let $P \in \mathcal{P}$ and $Q \in \mathcal{P}$, then the contaminated distribution P^λ is defined for all $\lambda \in [0, 1]$ by

$$P^\lambda = (1 - \lambda)P + \lambda Q.$$

We denote extreme value function and optimal set mapping of contaminated stochastic programming problem as

$$\varphi(\lambda) = \inf_{x \in X} g(x, P^\lambda), \quad (2)$$

$$\psi(\lambda) = \arg \min_{x \in X} g(x, P^\lambda) = \{x \in X : g(x, P^\lambda) = \varphi(\lambda)\}.$$

The Gateaux derivative of the extreme value function at P in direction $Q - P$ is then defined as

$$\varphi'(P; Q - P) = \lim_{\lambda \rightarrow 0+} \frac{\varphi(\lambda) - \varphi(0)}{\lambda}.$$

If we assume, that both optimal values $\varphi(0), \varphi(1)$ are finite and the derivative $\varphi'(P; Q - P)$ exists, the concavity of the objective function in the underlying distribution ensures concavity of the extreme value function. Hence, we can construct the contamination bounds for the extreme value function of the contaminated problem (2) as follows

$$(1 - \lambda)\varphi(0) + \lambda\varphi(1) \leq \varphi(\lambda) \leq \varphi(0) + \lambda\varphi'(P; Q - P), \\ \lambda \in [0, 1].$$

In order to evaluate these bounds we need to evaluate the Gateaux derivative of the optimal value function or at least upper bound for the derivative. However, we do not need to solve any contaminated problem which is always larger than the original and fully contaminated problem.

III. MEAN-CVaR MODEL

A. Value at Risk and Conditional Value at Risk

In this part we review definitions of Value at Risk and Conditional Value at Risk and mention their basic properties and relations, cf. [17].

If we denote Z a general loss variable with distribution function F , then α VaR is defined as

$$VaR_\alpha = \min\{z : F(z) \geq \alpha\}$$

for some level $\alpha \in (0, 1)$, usually 0.95 or 0.99. We must be careful when we define CVaR. The popular definition as "mean of losses greater than VaR" is inaccurate in general, see [17]. Correctly, CVaR is defined as mean of losses in the α -tail distribution

$$F_\alpha(z) = \begin{cases} \frac{F(z) - \alpha}{1 - \alpha}, & \text{if } z \geq VaR_\alpha \\ 0, & \text{otherwise.} \end{cases}$$

In general, it can even hold, see [17]:

$$\mathbb{E}[Z|Z \geq VaR_\alpha] < CVaR_\alpha < \mathbb{E}[Z|Z > VaR_\alpha].$$

For application of CVaR in optimization problems, the following minimization formula is of crucial importance, cf. [17, Theorem 10]:

$$CVaR_\alpha = \min_{\eta \in \mathbb{R}} \left[\eta + \frac{1}{1 - \alpha} \mathbb{E}[Z - \eta]^+ \right], \quad (3)$$

TABLE I
TRANSACTION COSTS

CZK			l	a_l	c_l
0 - 10 000	2 %		0	0	0
10 000 - 50 000	1.5 %	+	1	10 000	150
50 000 - 250 000	1 %	+	2	50 000	800
250 000 - 500 000	0.5 %	+	3	250 000	2800
500 000 and more	0 %	+	4	500 000	4050

where $[\cdot]^+$ denotes positive part and η is a real auxiliary variable. The set of optimal solutions is a closed interval, which contains VaR_α as its lower boundary, possibly reducing exactly to VaR_α . If the loss variable depends on decision variables, say $Z(x)$, $x \in X$, minimization of CVaR can be converted into minimization of the auxiliary function defined in (3) over the auxiliary variable η and simultaneously over decisions x , i.e.

$$\min_{x \in X} CVaR_\alpha(x) = \min_{(\eta, x) \in \mathbb{R} \times X} \left[\eta + \frac{1}{1-\alpha} \mathbb{E}[Z(x) - \eta]^+ \right].$$

The previous relation is called optimization shortcut, cf. [17].

B. Mean-risk model

In this part we formulate investment problem with transaction costs, integer allocations and CVaR risk measure. Similar investment problems with CVaR and MAD risk measures and transaction costs were discussed in [4], [14].

We denote P_i quotation of security i , f_i fixed transaction costs, c_i proportional transaction costs (not depending on investment amount), R_i random return of security i , x_i number of securities, y_i binary variables which indicate, whether the security i is bought or not. Then the loss random function depending on our decision x, y and random returns R is equal to

$$Z(x, y, R) = - \sum_{i=1}^n (R_i - c_i) P_i x_i + \sum_{i=1}^n f_i y_i$$

together with the constraints $0 \leq x_i \leq u_i y_i$ using upper bounds $u_i > 0 \forall i$.

Typical real proportional transaction cost are not constant but depend on invested amount. We denote such cost function $c_i(P_i x_i)$, which is usually piecewise linear concave, see Fig. 1, and can be given by the points $(a_{il}, c_{il})_{l=0}^{L_i}$, where $a_0 < \dots < a_L$ are bounds of intervals and $c_0 < \dots < c_L$ are corresponding transaction costs, see Table I. If we set the highest admissible investment into the asset i to 500 000 CZK, the cost function can be rewritten using linear functions

and auxiliary binary variables.¹

$$\begin{aligned} c(Px) &= \sum_{l=0}^L \gamma_l c_l, \quad Px = \sum_{l=0}^L \gamma_l a_l, \\ \gamma_0 &\leq y_0, \\ \gamma_l &\leq y_l + y_{l-1}, \quad l = 1, \dots, L-1, \\ \gamma_L &\leq y_L, \\ \sum_{l=0}^L y_l &= 1, \quad y_l \in \{0, 1\}, \\ \sum_{l=0}^L \gamma_l &= 1, \quad \gamma_l \geq 0. \end{aligned}$$

Other possibility is that the transaction costs are piecewise constant depending on investment amount, see [14].

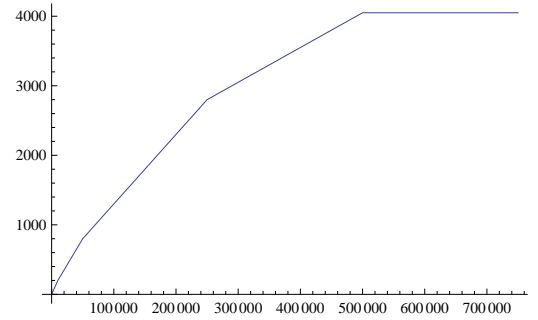


Fig. 1. Concave transaction costs from Table I (nondifferentiable points $(a_l, c_l)_{l=0}^4$)

We will assume that the distribution of random returns is finite discrete, i.e. $P \sim D(\{p_j, r_j^P\}_{j=1}^{J^P})$ with probabilities $p_j \geq 0$ of realizations r_j^P , and $\sum_{j=1}^{J^P} p_j = 1$.

To be able to apply contamination techniques directly we need to have objective function of expectations type and no random parts in constraints. This corresponds to the aggregate function approach, i.e. the objective function of our problem is given by

$$g_\rho(\eta, x, y; P) = (1-\rho) \sum_{j=1}^J p_j Z(x, y, r_j^P) \quad (4)$$

$$+ \rho \left(\eta + \frac{1}{1-\alpha} \sum_{j=1}^J p_j [Z(x, y, r_j^P) - \eta]^+ \right), \quad (5)$$

where $\rho \in (0, 1)$ is a parameter corresponding to the aggregate function. If we set $\rho = 0$ we minimize expected loss without involving risk minimization. On the other hand, if we set $\rho = 1$ we are absolutely risk averse, i.e. we minimize risk

¹For simplicity we drop the index i .

only without considering mean loss (return). Our investment problem is

$$\begin{aligned} \min \quad & g_\rho(\eta, x, y; P) \\ \text{s.t.} \quad & l_i y_i \leq x_i \leq u_i y_i, \quad i = 1, \dots, n, \\ & C_l \leq \sum_{i=1}^n P_i x_i \leq C_u, \\ & x_i \geq 0, \text{ integer}, \quad i = 1, \dots, n, \\ & y_i \in \{0, 1\}, \quad i = 1, \dots, n, \\ & \eta \in \mathbb{R}, \end{aligned} \quad (6)$$

where C_l and C_p are lower and upper bound on the capital available for the portfolio investment, $l_i > 0$ and $u_i > 0$ are lower and upper number of units for each security i . Using auxiliary variables, the investment problem can be reformulated as mixed-integer linear programming problem.

$$\begin{aligned} \min \quad & \left[(1 - \rho) \sum_{j=1}^J p_j \left(- \sum_{i=1}^n (r_{ij} - c_i) P_i x_i + \sum_{i=1}^n f_i y_i \right) \right. \\ & \left. + \rho \left(\eta + \frac{1}{1-\alpha} \sum_{j=1}^J p_j v_j \right) \right] \\ \text{s.t.} \quad & - \sum_{i=1}^n (r_{ij} - c_i) P_i x_i + \sum_{i=1}^n f_i y_i - \eta \leq v_j, \\ & j = 1, \dots, J, \\ & l_i y_i \leq x_i \leq u_i y_i, \quad i = 1, \dots, n, \\ & C_l \leq \sum_{i=1}^n P_i x_i \leq C_u, \\ & v_j \geq 0, \quad j = 1, \dots, J, \\ & x_i \geq 0, \text{ integer}, \quad i = 1, \dots, n, \\ & y_i \in \{0, 1\}, \quad i = 1, \dots, n, \\ & \eta \in \mathbb{R}. \end{aligned}$$

Cardinality constraints on maximal number of different assets m^a in a portfolio can be formulated using the binary variables. If our assets can be split into different sectors, we may have cardinality constraints in any sector, say $i = n_1, \dots, n_2$, on maximal number of different assets m^s , i.e.

$$\sum_{i=n_1}^{n_2} y_i \leq m^s.$$

On the other hand, if we want to buy at least one asset in the sector, we set

$$\sum_{i=n_1}^{n_2} y_i \geq 1.$$

If you buy asset i_1 , you can not buy i_2 , can be expressed as a constraint

$$y_{i_2 t} (\xi^{t-1}) \leq (1 - y_{i_1 t}). \quad (7)$$

It is necessary to notice that all mentioned constraints are linear. We can also incorporate several institutional constraints in similar way etc.

C. Contaminated problem and contamination bounds

We consider another finite discrete distribution $Q \sim D(\{q_j, r_j^Q\}_{j=1}^{J^Q})$ with probabilities $q_j \geq 0$ of realizations r_j^Q , and $\sum_{j=1}^{J^Q} q_j = 1$, which will be used as contamination distribution. We denote the optimal value function $\varphi(\lambda)$ and the set of optimal solutions

$$\begin{aligned} \psi(\lambda) &= \{(\eta, x, y) \in \mathbb{R}^{n+1} \times \{0, 1\}^n : \\ & (1 - \lambda) g_\rho(\eta, x, y; P) + \lambda g_\rho(\eta, x, y; Q) = \varphi(\lambda)\} \end{aligned}$$

of the contaminated problem

$$\begin{aligned} \varphi(\lambda) &= \min \quad (1 - \lambda) g_\rho(\eta, x, y; P) + \lambda g_\rho(\eta, x, y; Q) \\ \text{s.t.} \quad & l_i y_i \leq x_i \leq u_i y_i, \quad i = 1, \dots, n, \\ & C_l \leq \sum_{i=1}^n P_i x_i \leq C_u, \\ & x_i \geq 0, \text{ integer}, \quad i = 1, \dots, n, \\ & y_i \in \{0, 1\}, \quad i = 1, \dots, n, \\ & \eta \in \mathbb{R}, \lambda \in [0, 1]. \end{aligned} \quad (8)$$

We assume that both the original problem (for $\lambda = 0$) and the fully contaminated problem (for $\lambda = 1$) have nonempty set of optimal solutions. For the directional derivative of the extreme value function the following equalities hold, cf. [6]:

$$\begin{aligned} \varphi'_+(0) &= \min_{\psi(0)} g_\rho(\eta, x, y; Q) - \varphi(0), \\ \varphi'_-(1) &= \min_{\psi(1)} g_\rho(\eta, x, y; P) - \varphi(1). \end{aligned}$$

Using the explicit formulas for directional derivatives we can construct upper contamination bounds.

$$\begin{aligned} (1 - \lambda) \varphi(0) + \lambda \varphi(1) &\leq \varphi(\lambda) \\ &\leq \min\{\varphi(0) + \varphi'_+(0) \lambda, \varphi(1) + \varphi'_-(1) (1 - \lambda)\}, \\ \lambda &\in [0, 1]. \end{aligned}$$

Note, that a solution of the original and the fully contaminated problem is needed to obtain the bounds.

IV. NUMERICAL STUDY

We would like to invest 500 000 CZK into Czech shares funds using Mean-CVaR model with real features introduced in previous chapter. We will also demonstrate practical use of contamination techniques.

We consider 30 Czech shares funds which can be divided into 4 types:

- *stock funds* ($i \in \{1, \dots, 8\}$): ČPI - OPF global.znacek, ČSOB Akciovy Mix, IKS Svetovych indexu, ISČS-SPOROTREND, Pioneer akciovy fond, ČPI - OPF Fond nove ekonomiky, ČPI - OPF Fond ropneho a energetiky, ČPI - OPF Fond farmacie a biotec;
- *bond funds* ($i \in \{9, \dots, 14\}$): ČPI - OPF korp. dluhopisu, ČSOB Bond Mix, ISČS-BONDINVEST,

ISČS-SPOROBOND, ISČS-TREND BOND, Pioneer obligacni fond;

- *financial funds* ($i \in \{15, \dots, 18\}$): ČPI - OPF Penezni, IKS Penezni trh, ISČS-SPOROINVEST, Pioneer Sporokonto;
- *mixed funds* ($i \in \{19, \dots, 30\}$): ČPI - Smiseny OPF, ČSOB bohatstvi, ČSOB nadacni, ČSOB stredoevropsky, IKS Balancovany, IKS Global konzervativni, ISČS-Dynamicky Mix FF, ISČS-FOND RIZENYCH VYNOSU, ISČS-Vynosovy OPF, ISČS-Vyvazeny Mix FF, Pioneer dynamicky fond, Pioneer rustovy fond.

Week returns r^w from January 2005 to April 2009 were downloaded from [2]. We used four successive returns to estimate month returns, i.e. $r^m = \prod_{t=1}^4 (1 + r_t^w) - 1$, on which we based our portfolio optimization model. We also include two riskless assets (term deposits) into our model ($i \in \{31, 32\}$). We use condition similar to (7) to differ two zones with different guaranteed interest rates, i.e. if you deposit between 50 000 CZK and 250 000 CZK, your interest will be lower than if you deposit between 250 000 CZK to 500 000 CZK. Proportional transaction costs range from 0 to 2 per cent depending on the fund.

In the source of data we can distinguish two periods - before and during distress. We use the first period to construct our portfolio and the second period to the post-analysis of our results. We study performance of our portfolios and apply the contamination technique. We choose four portfolios with different risk-aversion parameters ρ , see Table II, and test their in-sample, Fig. 2, and out-of-sample performance, Fig. 3. We see that risky portfolio ($\rho = 0$) behaves well during in-sample period, however during out-of-sample period leads to the greatest loss. On the other hand, the most conservative portfolio ($\rho = 1$) brings the least losses during distress. Contamination bounds, see Fig. 4, show that the portfolio risk increases with higher contamination.

The numerical study was processed on HP PC with Intel(R) Pentium(R) Dual CPU E2200 @ 2.20Ghz, 3GB RAM, 300GB hard disk and Windows Vista Home Premium system. MPL modelling system [3] and CPLEX solver [1] were used to solve the problems, which do not take more than few seconds.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

We introduced investment problem with integer allocations, transaction costs and Conditional Value at Risk as a risk measure. We proposed contamination techniques, which enabled us to quantify the change in optimal value, if the underlying distribution was contaminated by another distribution.

TABLE II
INVESTMENTS (IN THOUSANDS CZK)

ρ	0	0.4	0.5	1
ISČS-SPOROTREND	100	30	0	0
ČSOB Bond Mix	0	70	100	0
ČPI - OPF Penezni	100	100	100	100
Pioneer Sporokonto	100	100	100	0
IKS Balancovany	100	100	100	50
IKS Global konzervativni	100	100	100	100
Term deposits	0	0	0	250
Expected loss	-776.55	-553.08	-485.87	-288.58
$CVaR_\alpha$	472.43	48.19	-27.54	-31.50

TABLE III
CONTAMINATION FOR $\rho = 1$

Distribution	P	Q
ČPI - OPF Penezni	100 000 CZK	100 000 CZK
IKS Balancovany	50 000 CZK	0
IKS Global konzervativni	100 000 CZK	0
Term deposits	250 000 CZK	400 000 CZK
$CVaR_\alpha$	-31.5020	189.1228
VaR_α	-4572.7500	-394.6000
ϕ'	2579.9976	-95.1548

We applied introduced investment model to real data of 30 Czech investment funds. We studied in-sample and out-of-sample performance of portfolios with different risk aversions and applied contamination techniques to study the behaviour of the risk before and during distress.

B. Future Works

First computational experiments show that solving multi-period investment problems is much more computationally demanding. Without any special approaches it leads to solving large scale mixed-integer problems which might take huge number of time. It will be necessary to take into account structure of the problem and to use decomposition algorithms, see [20], which are able to decompose the large scale original problems into smaller ones. The future research will be devoted to this area.

VI. ACKNOWLEDGMENTS

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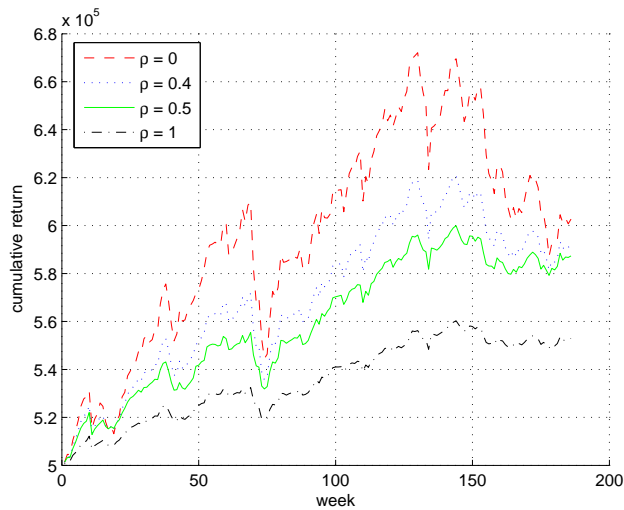


Fig. 2. In-sample portfolio value

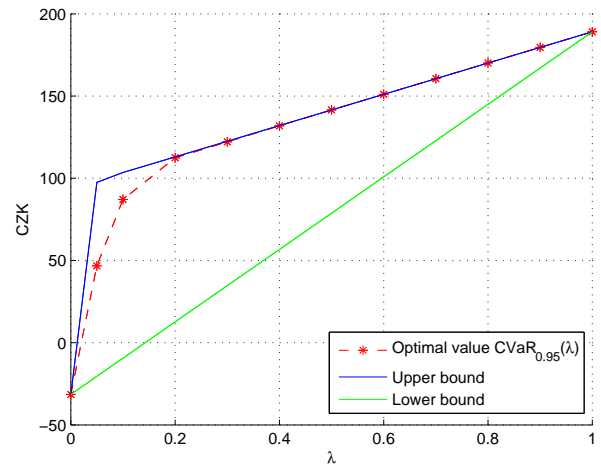


Fig. 4. Contamination bounds

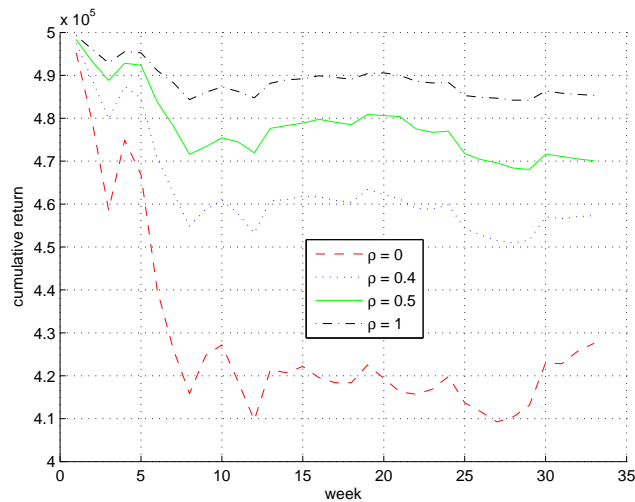


Fig. 3. Out-of-sample portfolio value

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