Predictive Maintenance using Dynamic Probabilistic Networks

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Abstract

We study dynamic reliability of systems where system components age with a constant failure rate and there is a budget constraint. We develop a methodology to effectively prepare a predictive maintenance plan of such a system using dynamic Bayesian networks (DBNs). DBN representation allows monitoring the system reliability in a given planning horizon and predicting the system state under different replacement plans. When the system reliability falls below a predetermined threshold value, component replacements are planned such that maintenance budget is not exceeded. The decision of which component(s) to replace is an important issue since it affects future system reliability and consequently the next time to do replacement. Component marginal probabilities given the system state are used to determine which component(s) to replace. Two approaches are proposed in calculating the marginal probabilities of components. The first is a myopic approach where the only evidence belongs to the current planning time. The second is a look-ahead approach where all the subsequent time intervals are included as evidence.

1 Introduction

Maintenance can be performed either after the breakdown takes place or before the problem arises. The former is reactive whereas the latter is proactive. Reactive maintenance is appropriate for systems where the failure does not result in serious consequences. Decision-theoretic troubleshooting belongs to this category. Proactive or planned maintenance can be further classified as preventive and predictive (Kothamasu et al., 2006). These differ in the scheduling behaviour. Preventive maintenance performs maintenance on a fixed schedule whereas in predictive maintenance, the schedule is adaptively determined. Reliability-centered maintenance is predictive maintenance where reliability estimates of the system are used to develop a cost-effective schedule.

Decision-theoretic troubleshooting, which balances costs and likelihoods for the best action, is first studied by (Kalagnanam and Henrion, 1988). Heckerman et al.(1995) extend it to the context of Bayesian networks (Pearl, 1988). A similar troubleshooting problem, where multiple but independent faults are allowed, is addressed in (Srinivas, 1995). More recent studies are mostly due to researchers from the SACSO (Systems for Automated Customer Support Operations) project. By assuming a single fault, independent actions and constant costs, and making use of a simple model representation technique (Skaanning et al., 2000), they show that the simple efficiency ordering yields an optimal sequence of actions (Jensen et al., 2001). Langseth and Jensen (2001) present two heuristics for handling dependent actions and conditional costs. Langseth and Jensen (2003) provide a formalism that combines the methodologies used in reliability analysis and decision-theoretic troubleshooting. Koca and Bilgiç (2004) present a generic decision-theoretic troubleshooter to handle troubleshooting tasks incorporating questions, dependent actions, conditional costs, and any combinations of these. Decision-theoretic troubleshooting has always

been studied as a static problem and with an objective to reach a minimum-cost action plan.

On the reliability analysis side, fault diagnosis finds its roots in (Vesely, 1970). Kothamasu et al.(2006) review the philosophies and techniques that focus on improving reliability and reducing unscheduled downtime by monitoring and predicting machine health. Torres-Toledano and Sucar (1998) develop a general methodology for reliability modelling of complex systems based on Bayesian networks. Welch and Thelen (2000) apply DBNs to an example from the dynamic reliability community. Weber and Jouffe (2003, 2006) present a methodology that helps developing DBNs and Dynamic Object Oriented Bayesian Networks (DOOBNs) from available data to formalize reliability of complex dynamic models. Muller et al. (2004) propose a methodology to design a prognosis process taking into account the behavior of environmental events. They develop a probabilistic model based on the translation of a preliminary process model into DBN. Bouillaut et al. (2004) use causal probabilistic networks for the improvement of the maintenance of railway infrastructure. Weber et al. (2004) use DBN to model dependability of systems taking into account degradations and failure modes governed by exogenous constraints.

All of the studies related to reliability analysis using DBNs are *descriptive*. Dynamic problem is represented with DBNs and the outcome of the analysis is how system reliability behaves in time. The impact of maintenance of an element at a specific time on this behaviour is also reported in some of them. However optimization of maintenance activities (i.e., finding a minimum cost plan) is not considered which is the main motivation of our paper. Maintenance is expensive and critical in most systems. Unexpected breakdowns are not tolerable. That is why planning maintenance activities intelligently is an important issue since it saves money, service time and also lost production time. In this study, we are trying to optimize maintenance activities of a system where components age with a constant failure rate and there is a budget constraint. We develop a methodology to effectively prepare a predictive maintenance plan using DBNs. One can argue that the failure of the system and its associated costs can be modelled using influence diagrams or limited memory influence diagrams (LIMIDs). But we propose a way of representing the problem as an optimization problem first and then use only DBNs for fast inference under some simplifying assumptions.

The rest of the paper is organized as follows: In Section 2, problem is defined and in Section 3, dynamic Bayesian network based models are proposed as a solution to the problem defined. Two approaches are presented in scheduling maintenance plans. Numerical results are given in Section 4. Finally Section 5 gives conclusions and points future work.

2 Problem Definition

The problem we take up can be described as follows: There is a system which consists of several components. We observe the system in discrete epochs and assume that system reliability is observable. System reliability is a function of the interactions of the system components which are not directly observable. We presume that the reliability of the system should be kept over a predetermined threshold value in all periods. This is reasonable in mission critical systems where the failure of the system is a very low probability event due to built in redundancy and other structural properties. Therefore, we do not explicitly model the case where the system actually fails. Components age with a constant rate and it is possible to replace components in any period. Once replaced, the components will work at their full capacity. There is a given maintenance budget for each period, which the total replacement cost in that period cannot exceed. Our aim is to minimize total maintenance cost in a planning horizon such that reliability of the system never falls below the threshold and maintenance budget is not exceeded. Furthermore we make the following assumptions:

(i) Lifetime of any component in the system is exponentially distributed. That is failure rate

of any component is constant for all periods.

(ii) All other conditional probability distributions used in the representation are discrete.

(iii) All components and the system have two states ("1" is the working state, "0" is the failure state).

(iv) Components can be replaced at the beginning of each period. Once they are replaced, their working state probability (i.e., their reliability) become 1 in that period.

The problem can be expressed as a mathematical optimization problem.

The model parameters are:

i : index of components
t : index of time periods
λ_i : failure rate of component i
R_{i1} : initial reliability of component i
c_{it} : cost of replacing component i in period t
B_t : available maintenance budget in period t
L : threshold value of system reliability
$f(\cdot)$: function mapping component reliabilities R_{it} to system reliability R_{0t}

The decision variables are:

$$X_{it} = \begin{cases} 1 & \text{if component } i \text{ is replaced in period } t \\ 0 & \text{otherwise} \end{cases}$$

R_{it} : reliability of component i in period t
R_{0t} : reliability of system in period t

The Predictive Maintenance (PM) model can be formulated mathematically as follows:

$$Z(PM) = \min \sum_{t=1}^{T} \sum_{i=1}^{n} c_{it} X_{it} \tag{1}$$

subject to

$$\sum_{i=1}^{n} c_{it} X_{it} \leq B_t, \ \forall t \tag{2}$$

$$R_{0t} \geq L, \ \forall t \tag{3}$$

$$R_{it} = (1 - X_{it}) e^{-\lambda_i} R_{i,t-1} + X_{it}, \ \forall i, t \tag{4}$$

$$R_{0t} = f(R_{1t}, R_{2t}, ...R_{nt}), \ \forall t \tag{5}$$

$$X_{it} \in \{0, 1\}, \ \forall i, t \tag{6}$$

$$0 \leq R_{it} \leq 1, \ \forall i, t \tag{7}$$

The objective function (1) aims to minimize the total component replacement cost. Constraint set (2) represents the budget constraints. Constraint set (3) guarantees that system reliability in each period should be greater than the given threshold value. Constraints in (4) ensure that if components are replaced their reliability becomes 1, otherwise it will decrease with corresponding failure rates. System reliability in each period is calculated by constraint (5). Finally constraints (6) and (7) define the bounds on decision variables. In general, solving this problem may be quite difficult. The difficulty lies in constraint sets (4) and (5). Constraint set (4) defines a non-linear relation of the decision variables whereas constraint set (5) is much more generic. In fact, system reliability at time t can be a function of whole history of the system. It is this set of constraints and the implied relationships of constraint set (4) that we represent using a dynamic Bayesian network.

Further assumptions are imposed in order to simplify the above problem:

(v) Replacement costs of all components in any period are the same and they are all normalized at one.

(vi) Available budget in any period is normalized at one.

These assumptions indicate that in any period, only one replacement can be planned and the objective function we are trying to minimize becomes the total number of replacements in a planning horizon.

3 Proposed Solution

The mathematical problem may be solved analytically or numerically once the constraint set (5) is made explicit. However, as the causal relations (represented with constraint set (5) in the problem formulation) between the components and the system becomes more complex, it gets difficult to represent and solve it mathematically. We represent the constraint set (5) using dynamic Bayesian Networks (DBNs) (Murphy, 2002). A DBN is an extended Bayesian network (BN) which includes a temporal dimension. BNs are a widely used formalism for repre-

senting uncertain knowledge. The main features of the formalism are a graphical encoding of a set of conditional independence relations and a compact way of representing a joint probability distribution between random variables (Pearl, 1988). BNs have the power to represent causal relations between components and the system using conditional probability distributions. It is possible to analyse the process over a large planning horizon with DBNs.

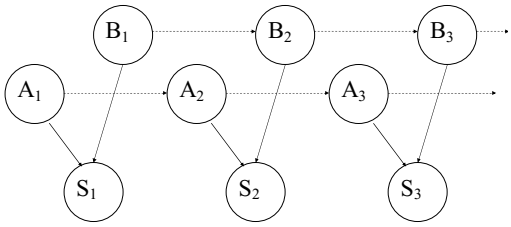

Figure 1: DBN representation of a system with 2 components

Figure 1 illustrates a DBN representation of a system with 2 components, A and B. Solid arcs represent the causal relations between the components and the system node whereas dashed arcs represent temporal relation of the components between two consecutive time periods. Note that this representation is Markovian whereas DBNs can represent more general transition structures. Temporal relations are the transition probabilities of components due to aging. Since the lifetime of any component in the system is exponentially distributed, the transition probabilities are constant because of the memoryless property of the exponential distribution given the time intervals are equal. Transition probability table for a component with two states ("1" is the working state, "0" is the failure state) is given in Table 1.

Table 1: Transition probability for component i

	Comp(t)	
Comp($t+1$)	1	0
1	$e^{-\lambda_i \triangle t}$	0
0	1-$e^{-\lambda_i \triangle t}$	1

DBN representation allows monitoring the system reliability in a given planning horizon and predicting the system state under different replacement schedules. When the system reliability falls below a predetermined threshold value, a component replacement is planned. The decision of which component to replace is an important issue since it affects future system reliability and consequently the next time to do a replacement. Like in decision-theoretic troubleshooting (Heckerman et al., 1995), marginal probabilities of components given the system state are used as efficiency measures of components in each period when a replacement is planned. Let S_k the denote system state in period k and C_{ik} denote the state of component i in period k. The following algorithm summarizes our DBN approach:

(i) Initialize $t = 1$
(ii) Infer system reliability $P(S_k = 1)$ $t \leq k \leq T$
(iii) Check if $P(S_k = 1) \leq L$
(iv) If $P(S_k = 1) \geq L$ $\forall k$, then stop.
(v) Else prepare a replacement plan for period k
  (a) Calculate $P_{ik} = P(C_{ik} = 0 | S_k = 0)$ $\forall i$
  (b) $i^* = \arg\max\{P_{ik}\}$
  (c) Update reliability of i^* in k to 1.
    $P(C_{ik} = 1) \leftarrow 1$
  (d) Update $t = k + 1$
(vi) If $t > T$, then stop.
(vii) Else continue with step (ii)

Note that $P(S_t = 1) = R_{0t}$ and $P(C_{it} = 1) = R_{it}$. This is a *myopic approach*, since the only evidence in calculating marginal probabilities in (v.a) belongs to the system state at the current planning time. An alternative approach is to take into account future information which can be transmitted by the transition probabilities of components. This is done by entering evidence to the system node from $k + 1$ to T as $S_{k+1:T} = 0$. We call this approach the *look-ahead approach*. The algorithm is the same as above except for step (v.a) which is replaced as follows in look-ahead approach:

(v.a) Calculate $P_{ik} = P(C_{ik} = 0 | S_{k+1:T} = 0)$ $\forall i$ where $S_{k+1:T}$ denotes $S_{k+1}, ..., S_T$.

4 Numerical Results

The DBN algorithm is coded in Matlab and uses the Bayes Net Toolbox (BNT) (Murphy, 2001) to represent the causal and temporal relations, and to infer the reliability of the system. Two approaches, myopic and look-ahead, are compared on a small example with two components given in Figure 1. The planning horizon is taken as 100 periods and the threshold value is given as 0.50.

First scenario is created by taking mean time to failure (MTTF) $(1/\lambda_i)$ of each component i equal which is set at 40 periods. The same replacement plan, given in Figure 2, is generated by both approaches. This is because components have equal MTTFs and hence equal transition probabilities. System reliability of scenario 1 is illustrated in Figure 2 where the peak points are the periods where a component is replaced. On each peak point, the component which is planned for replacement is indicated in the figure. 4 replacements are planned in both approaches. When a replacement occurs, system reliability jumps to a higher reliability value, and then gradually decreases as time evolves until the next replacement.

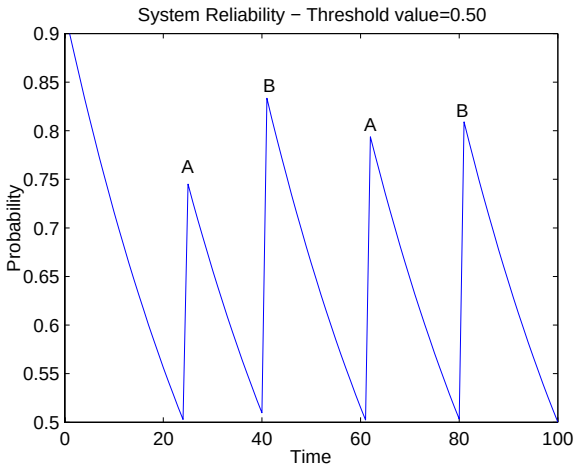

Figure 2: Scenario 1- system reliability and replacement plan

Second scenario is created by differentiating MTTFs of components. We decrease MTTF of component B to 10 periods. Replacement plans in Figure 3 and Figure 4 are generated by the myopic and look-ahead approaches, respectively. Replacement plans of the two approaches differ since components have different transition probabilities. Both approaches begin their replacement plan in period 12, by selecting the same component to replace. In the next replacement period ($k = 21$), different components are selected. Myopic approach selects component B, because this replacement will make the system reliability higher in the short-term. Look-ahead approach selects component A, because this replacement will make the system reliability higher in the long-run. This is further illustrated in Table 2. Although system reliability in the myopic approach is higher at $t = 21$, it decreases faster than the system reliability under the look-ahead approach. Hence, the look-ahead approach plans its next replacement at $t = 29$ while the myopic approach plans its next replacement at $t = 28$. By selecting A instead of B, the look-ahead approach defers its next replacement time. So as a total, in scenario 2, the look-ahead approach generates 10 replacements in 100 periods while the myopic approach generates 11.

Table 2: Scenario 2- system reliability where $21 \leq t \leq 28$

Period	21	22	23	24
Myopic	.7712	.7169	.6673	.6220
Look-ahead	.7036	.6718	.6421	.6144
Period	25	26	27	28
Myopic	.5805	.5425	.5077	.4758
Look-ahead	.5884	.5642	.5414	.5200

System reliability of scenario 2 is illustrated in Figures 3 and 4 for myopic and look-ahead approaches, respectively. In Figure 3, there are 11 peak points which means 11 replacements are planned. In Figure 4, there are 10 peak points which means 10 replacements are planned.

A third scenario is also carried out by further decreasing MTTF of component B to 5 periods. The discrepancy between plans generated by the two approaches becomes more apparent. Myopic approach plans a total of 19 replace-

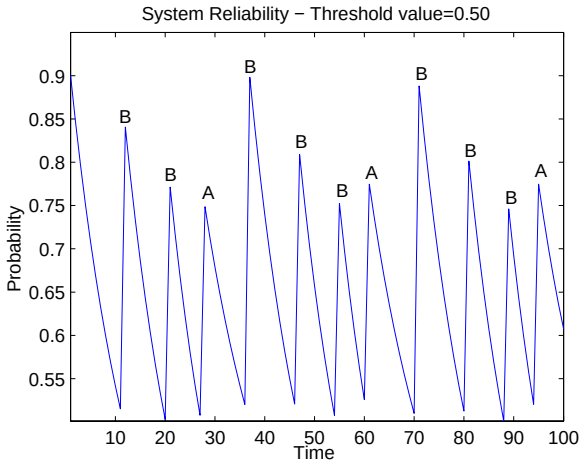

Figure 3: Scenario 2- system reliability and replacement plan with the myopic approach

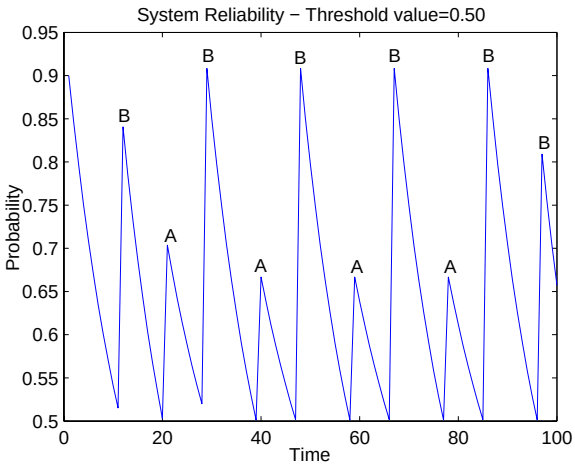

Figure 4: Scenario 2- system reliability and replacement plan with the look-ahead approach

ments while look-ahead approach plans a total of 16 replacements.

When the threshold value increases, an interesting question arises: Does it still worth to account for future information in choosing which component to replace? Table 3 shows the number of replacements found by the myopic and the look-ahead approaches at various threshold (L) values for scenario 2. When $L = 0.80$, myopic approach finds fewer replacements than the look-ahead approach. This is because as threshold increases, more frequent replacements will be planned, hence focusing on short-term reliability instead of the future reliability may result in less number of replacements.

Table 3: Number of replacements for the myopic and look-ahead approaches at various threshold values for $T = 100$

Threshold	Myopic	Look-ahead
0.50	11	10
0.60	15	15
0.70	23	22
0.80	33	35
0.90	67	67
0.95	100	100

In order to understand how good our methodology is, we enumerate all possible solutions of k replacements in a reasonable time horizon. Here, k refers to the upper bound of minimum replacements given by our DBN algorithm. The total number of solutions in a horizon of T periods with k replacements is given as:

$$\begin{pmatrix} T \\ k \end{pmatrix} 2^k \tag{8}$$

The first term is the total number of all possible size-k subsets of a size-T set. This corresponds to the total number of all possible time alternatives of k replacements in horizon T. The second term is the total number of all possible replacements for two components. Since this solution space becomes intractable with increasing T and k, a smaller part of the planning horizon is taken for enumeration of both scenarios. We started working with $T = 50$ periods where 2 repairs are proposed and $T = 30$ periods where 3 repairs are proposed by our algorithm for scenario 1 and scenario 2, respectively. The next replacements correspond to $t = 62$ (Figure 2) and $t = 40$ (Figure 4). Hence, we increased $T = 61$ and $T = 39$, the periods just before the 3rd and 4th replacements given by our algorithm for scenarios 1 and 2. The number of feasible solutions found by enumeration are reported in Table 4.

When we decrease k, number of replacements, by 1 ($k = 1$ and $k = 2$ for scenarios 1 and 2 re-

Table 4: Enumeration results

Scenario	Horizon	Number of Replacements	Feasible Solutions
1	50	2	324
1	50	1	0
1	61	2	2
1	61	1	0
2	30	3	1543
2	30	2	0
2	39	3	1
2	39	2	0

spectively); no feasible solution is found. So, it is not possible to find a plan with less replacements given by our algorithm for these cases. Note also that, in scenario 1 when $T = 61$ and $k = 2$, enumeration finds two feasible solutions which are in fact symmetric solutions found also by our algorithm. Similarly in scenario 2 when $T = 39$ and $k = 3$, enumeration finds one feasible solution which is the one found by our algorithm with the look-ahead approach. There are 1543 feasible solutions for scenario 2 with $T = 30$ and $k = 3$. Our lookahead approach finds one of these solutions and the solution it finds has the maximum system reliability at $T = 30$ among all solutions. The same observation is also valid for scenario 1 with $T = 50$ and $k = 2$.

5 Conclusion

We study dynamic reliability of a system where components age with a constant failure rate and there is a budget constraint. We develop a methodology to effectively prepare a good predictive maintenance plan using DBNs to represent the causal and temporal relations, and to infer reliability values. We try to minimize number of component replacements in a planning horizon by first deciding the time and then the component to replace. Two approaches are presented to choose the component and they are compared on three scenarios and various threshold values. When failure rates of components are equal, they find the same replacement plan. However, as failure rates differ, the two approaches may end up with different number of replacements. This is because the look-ahead approach takes future system reliability into consideration while the myopic approach focuses on the current planning time.

In this kind of predictive maintenance problem, there are two important decisions: One is the time of replacement. Replacement should be done such that system reliability is always guaranteed to be over a threshold. The other decision is which component(s) to replace in that period such that budget is not exceeded and total replacement cost is minimized. Our methodology is based on separating these decisions under assumptions (v) and (vi). We give the former decision by monitoring the first period when system reliability just falls below a given threshold. In other words, we defer a replacement decision as far as the threshold permits. As for the latter decision, we propose two approaches, myopic and look-ahead, to choose the component to replace. By enumerating feasible solutions in a reasonable horizon, we show that our method is effective for our simplified problem where the objective has become minimizing total replacements in a given planning horizon.

In this paper, we outline a method that can be used for finding a minimum-cost predictive maintenance schedule such that the system reliability is always above a certain threshold. The approach is normative in nature as opposed to descriptive which is the case in most of the literature that uses DBNs in reliability analysis.

The problem becomes more complex by (i) differentiating component costs (in time), (ii) differentiating available budget in time, (iii) defining a maintenance fixed cost for each period which may or may not differ in time. In these cases, separating the two decisions may not give a good solution. Studying such cases is left for future work.

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