Sharing of probabilistic information of Bayesian agents

Bayesian agents

Bayesian agent
= set of random variables + joint domain + joint pdf

S agents

common set of variables	$(X_1,...,X_K) = \mathbf{X}$
common finite domain	$\{\mathbf{x}_1,...,\mathbf{x}_N\}$
pmfs (probability vectors)	$q_1,...,q_S$
true pmf	p

Supra Bayesian approach

scalar vectors $q_1,...,q_S,p$ vs. random vectors $\mathbf{q_1},...,\mathbf{q_S}, \mathbf{p}$
scalar matrix Q vs. random matrix **Q**

p (minus the last element) is continuously distributed
cpdf $t_{\mathbf{p}|Q}$ given $\mathbf{Q} = Q$

estimate **p**

minimize $E [K(\mathbf{p}||p) | \mathbf{Q} = Q]$

solution $p' = E_{t\mathbf{p}|Q} [\mathbf{p} | \mathbf{Q} = Q]$

Construction of $t_{\mathbf{p}|Q}$

principle of maximum entropy
principle of indifference wouldn't suffice

constraints $E_{t\mathbf{p}|Q}[\, K(\, q_s \,||\, \mathbf{p}\,) \,|\, \mathbf{Q} = Q\,] = I_s$

set of simultaneous implicit equations

Dirichlet distribution

$p' = w_0 q_0 + w_1 q_1 + \ldots + w_S q_S$

Nuance

$K (q_s \,||\, \mathbf{p})$ vs $K (\mathbf{p} \,||\, q_s)$
Kalenkovich vs Sechkarova

evaluation vs. compomise

union of supports vs. intersection

Extensions

- different supports
  - union
- generalized moments instead of pmfs
  - maximum entropy principle
- different supports + conditional marginal pmfs
  - (X_1,X_2,X_3), first agent - $q_{X2|X1}$
  - extend q_1
    - minimize $E [K(q_1 || \mathbf{p}) | \mathbf{Q} = Q]$
    - cpmf calculated from $q_1 = q_{X2|X1}$
    - $q_1 = p'_{X3|X2,X1} * q_{X2|X1} * p'_{X1}$ on the *support* of $q_{X2|X1}$
    - q_1 = p' elsewhere

Continuous case

S agents

common set of variables	$(X_1,...,X_K) = \mathbf{X}$
common bounded domain	A
pdfs	$g_1,...,g_S$
true pdf	g

random counterparts $\mathbf{g_1},...,\mathbf{g_S},\mathbf{g},\mathbf{G}$

the most entropic random process?

From continuous to discrete

the most entropic random process?

partition $(A_1,...,A_N)$ of A

$\mathbf{g} = I_{A1} * \mathbf{p}_1 / B(A_1) + ... I_{AN} * \mathbf{p}_N / B(A_N)$, $\mathbf{p}$, p

$t_{\mathbf{p}|Q}$, $E_{t\mathbf{p}|Q} [K (g_s \,||\, \mathbf{g}) \,|\, \mathbf{G} = G] = k_s$

arbitrary synthetic support $\{\mathbf{x}_1,...,\mathbf{x}_N\}$

$q_s(\mathbf{x}_n) = Int (q_s(\mathbf{x}) d\mathbf{x}, \mathbf{x} \text{ in } A_n)$

$I_s = k_s + Sum (B(A_n) q_s(x_n), n = 1,...,N)$

What is missing.

- I_s? where do they come from?
- better method for merging pdfs
- other, less important things